NYU MathUA-233 Theory of Probability (Summer 2017 Session II)

(Week 4) Mon July 24- Thurs July 27, 2017

1 Week 4 Topics: Chapter 5: Continuous Random Variables

1. Continuous random variable.
2. Probability density function (PDF) of a continuous random variable. CDF of a continuous random variable. Difference between PDF for a continuous random variable and PMF for discrete random variable.
3. Expectation, Variance, and moments of a random variable.
4. Moment generating function of a random variable.
5. Parameters of a continuous random variable (shape, location, etc for the PDF).
6. Expectation of a function of a random variable $g(X)$.
7. Probability density function of $g(X)$. Jacobian of the transformation $y = g(x)$.
8. Properties of the expectation.
9. Properties of the variance.
10. The exponential distribution, memoryless property. The gamma function $\Gamma(n)$.
11. The normal distribution (Gaussian) and the standard normal distribution $N(0, 1)$. De-Moivre Laplace limit theorem: normal approximation of the binomial distribution (after standarizing).
12. The uniform distribution $U(a, b)$ and generating other continuous random variables using $U(0, 1)$ (universality of the uniform distribution).
13. Other continuous random variables.

2 Reading assignment

Chapter 5 sections: 5.1 through 5.7.
Chapter 7 sections: 7.2 and 7.7.
Chapter 10 section: 10.2.

3 Homework Assignment

(Due Thursday July 27)

1. Page 212 (Problems): 5.1, 5.2, 5.11, 5.15, 5.24, 5.27, 5.32, 5.39, 5.40.
2. Page 215 (Theoretical problems): 5.21, 5.31.

(Due Tuesday August 1)

1. Let $X \sim N(0,1)$. Find $E|X|$.
2. Let $X \sim Poisson(\lambda)$. Find $E(X!)$.
3. Let $X \sim Geom(p)$ and let t be a constant. Compute the moment generating function $E(e^{tX})$ as a function of t.
4. Let U be a uniform random variable on the interval $(-1,1)$ (be careful about the minus signs).
   (a) Compute $E(U)$, $Var(U)$ and $E(U^4)$.
   (b) Find the CDF and PDF of U^2. Is the distribution of U^2 uniform on $(0,1)$?
5. (Universality of the uniform) Use the *universality of the uniform* $U(0,1)$ to present an approach for generating a random variable from the Weibull distribution $F(t) = 1 - e^{-at^{\beta}}$, $t \geq 0$.

4 Test 4 on Thursday July 27, 2017

Name:

1. (1 point) Consider the fourth central moment $\mu_4 = E((X-\mu)^4)$ for a given random variable X, where $E(X) = \mu$. Write an expression for μ_4 in terms of μ, $\mu_2' = E(X^2)$, $\mu_3' = E(X^3)$, and $\mu_4' = E(X^4)$.

2. (2 points) Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{8}x^{-1/2}e^{\frac{-x^{1/2}}{4}} & \text{for } x > 0, \\ 0 \text{ elsewhere.} \end{cases}$$

Calculate $E(X)$ and $Var(X)$. (Hint: you might need to use the gamma function $\Gamma(n)$.)

3. (2 points) Let $X \sim N(\mu, \sigma^2)$, prove that $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$, where $N(\mu, \sigma^2)$ is the normal distribution ($N(0,1)$ is the standard normal distribution).

4. (2 points) Suppose that the number of miles that a car can run before its battery wears out is exponentially distributed with an average value of $10,000$ miles. If a person desires to take a $5,000$ mile trip, what is the probability that he or she will be able to complete the trip without having to replace the battery? What can be said when the distribution is not exponential?

5. (2 points) In a large group of n people, find the *approximate* probability that at least 4 people end up with the same birthday. (Hint: you can define a Poisson random variable.)

6. (1 point) You have a random permutation of the integers $1, 2, \ldots, n$, where $n \geq 2$. Find the expected number of local minima (for example in $3, 2, 1, 4, 7, 5, 6$, a local minimum is any number that is smaller than its neighbors, in this case: 1 and 5 are local minima.)