MATH235 Calculus 1
Definitions

1. $\lim_{x\to c} f(x) = L$
For any $\epsilon > 0$, there exists some $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $0 < |x - c| < \delta$.

2. f is continuous at $x = c$.
$f(c)$ and $\lim_{x\to c} f(x)$ exist, and $f(c) = \lim_{x\to c} f(x)$.

3. f is differentiable at $x = c$.
$f'(c) = \lim_{h\to 0} \dfrac{f(c+h) - f(c)}{h}$ exists.

4. average rate of change/instantaneous rate of change of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$.

Let $x_2 = x_1 + h, h \neq 0$. Then, the average rate of change is $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(x_1 + h) - f(x_1)}{h}$, and the instantaneous rate of change is $\lim_{x_2\to x_1} \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \lim_{h\to 0} \dfrac{f(x_1 + h) - f(x_1)}{h}$.

5. f is increasing/decreasing on the interval (a, b).
f is increasing on (a, b) if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ for all x_1 and x_2 in (a, b).
f is decreasing on (a, b) if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ for all x_1 and x_2 in (a, b).

6. f is monotonic.
f is monotonic on the interval I if f is increasing on I or decreasing on I.

7. The graph of $y = f(x)$ is concave up/down on the interval (a, b).
f is concave up on (a, b) if f' is increasing on (a, b).
f is concave down on (a, b) if f' is decreasing on (a, b).

8. $\int_a^b f(x)dx$
Divide $[a, b]$ into n-subintervals: $[x_0, x_1], [x_1, x_2], [x_2, x_3], \cdots [x_{n-1}, x_n]$, where $x_0 = a$ and $x_n = b$. Let Δx_k be the width of the k-th subinterval, and let x_k^* be any point in the k-th subinterval, for all $1 \leq k \leq n$. Then

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x_k.$$