

MATH235 Calculus 1
Problems on Finding Derivatives.
09/23/2010

Find the derivative of the given function.

1. $y = 9 \tan \frac{x}{3}$

$$\begin{aligned} \frac{dy}{dx} &= 9 \sec^2\left(\frac{x}{3}\right) \frac{1}{3} && \text{by the Chain Rule,} \\ &= 3 \sec^2\left(\frac{x}{3}\right) \end{aligned}$$

2. $s = \sin^2(\theta^2 e^\theta) = (\sin(\theta^2 e^\theta))^2$

$$\begin{aligned} \frac{ds}{d\theta} &= 2 \sin(\theta^2 e^\theta) \frac{d}{d\theta}(\sin(\theta^2 e^\theta)) && \text{by the Power Rule and the Chain Rule,} \\ &= 2 \sin(\theta^2 e^\theta) \cos(\theta^2 e^\theta) \frac{d}{d\theta}(\theta^2 e^\theta) && \text{by the Chain Rule,} \\ &= 2 \sin(\theta^2 e^\theta) \cos(\theta^2 e^\theta) (2\theta e^\theta + \theta^2 e^\theta) && \text{by the Product Rule,} \\ &= 2\theta e^\theta \sin(\theta^2 e^\theta) \cos(\theta^2 e^\theta) (2 + \theta) \end{aligned}$$

3. $y = (9x^2 - 6x + 2)e^{x^3}$

$$\begin{aligned} \frac{dy}{dx} &= (18x - 6)e^{x^3} + (9x^2 - 6x + 2) \frac{d}{dx}e^{x^3} && \text{by the Product Rule,} \\ &= (18x - 6)e^{x^3} + (9x^2 - 6x + 2)e^{x^3} 3x^2 && \text{by the Chain Rule,} \\ &= e^{x^3} (27x^4 - 18x^3 + 6x^2 + 18x - 6) \end{aligned}$$

4. $r = \sec\sqrt{\theta} \tan \frac{1}{\theta}$

$$\begin{aligned} \frac{dr}{d\theta} &= \frac{d}{d\theta}(\sec\sqrt{\theta}) \tan \frac{1}{\theta} + \sec\sqrt{\theta} \frac{d}{d\theta}(\tan \frac{1}{\theta}) && \text{by the Product Rule,} \\ &= \sec\sqrt{\theta} \tan \sqrt{\theta} \left(\frac{d}{d\theta}\sqrt{\theta}\right) \tan \frac{1}{\theta} + \sec\sqrt{\theta} \sec^2 \frac{1}{\theta} \left(\frac{d}{d\theta} \frac{1}{\theta}\right) && \text{by the Chain Rule,} \\ &= \sec\sqrt{\theta} \tan \sqrt{\theta} \left(\frac{1}{2\sqrt{\theta}}\right) \tan \frac{1}{\theta} + \sec\sqrt{\theta} \sec^2 \frac{1}{\theta} \left(\frac{-1}{\theta^2}\right) \\ &= \sec\sqrt{\theta} \left(\frac{1}{2\sqrt{\theta}} \tan \sqrt{\theta} \tan \frac{1}{\theta} - \frac{1}{\theta^2} \sec^2 \frac{1}{\theta}\right) \end{aligned}$$

$$5. y = e^{\sqrt{3x+1}}$$

$$\frac{dy}{dx} = e^{\sqrt{3x+1}} \frac{d}{dx} \sqrt{3x+1}$$

by the Chain Rule,

$$= e^{\sqrt{3x+1}} \frac{1}{2} (3x+1)^{-\frac{1}{2}} \frac{d}{dx} (3x+1)$$

by the Power Rule and the Chain Rule,

$$= \frac{3e^{\sqrt{3x+1}}}{2\sqrt{3x+1}}$$

$$6. y = \sin^5 x = (\sin x)^5$$

$$\frac{dy}{dx} = 5 \sin^4 x \frac{d}{dx} \sin x$$

by the Chain Rule,

$$= 5 \sin^4 x \cos x$$

$$7. y = \frac{2}{3x-2} = 2(3x-2)^{-1}$$

$$\frac{dy}{dx} = -2(3x-2)^{-2} \frac{d}{dx} (3x-2)$$

by the Power Rule and the Chain Rule,

$$= \frac{-6}{(3x-2)^2}$$

$$8. y = 3x(x^2 + 2x)^{\frac{2}{3}}$$

$$\frac{dy}{dx} = 3(x^2 + 2x)^{\frac{2}{3}} + 3x \frac{d}{dx} ((x^2 + 2x)^{\frac{2}{3}})$$

by the Product Rule,

$$= 3(x^2 + 2x)^{\frac{2}{3}} + 3x \left(\frac{2}{3} (x^2 + 2x)^{-\frac{1}{3}} \frac{d}{dx} (x^2 + 2x) \right)$$

by the Power Rule and the Chain Rule,

$$= 3(x^2 + 2x)^{\frac{2}{3}} + \frac{4x^2 + 4x}{(x^2 + 2x)^{\frac{1}{3}}}$$

$$= \frac{3(x^2 + 2x) + 4x^2 + 4x}{(x^2 + 2x)^{\frac{1}{3}}}$$

$$= \frac{7x^2 + 10x}{(x^2 + 2x)^{\frac{1}{3}}}$$