MATH235 Calculus 1
Quiz 1

Show all work to receive full credit. Carefully write down your thought process. Your solution must not contain any logical errors. Good luck!

1. Solve the following inequality : $\dfrac{3}{x-1} < \dfrac{2}{x+1}$.

$$\frac{3}{x-1} - \frac{2}{x+1} < 0,$$
$$\frac{3(x+1)-2(x-1)}{(x-1)(x+1)} < 0,$$
$$\frac{x+5}{(x-1)(x+1)} < 0.$$

Notice that there are two possible ways that the above inequality can hold:

- case 1. $x+5>0$ and $(x-1)(x+1)<0 : -1<x<1$
- case 2. $x+5<0$ and $(x-1)(x+1)>0 : x<-5$

Therefore, the answer is $(-\infty,-5)\cup(-1,1)$.

2. Find the domain of the function $f(x)=\dfrac{\ln(x+1)}{\ln(x-2)}$.

Restrictions on the domain :

- $$\ln(x-2) \neq 0,$$
$$x \neq 3.$$

- $$x+1>0,$$
$$x>-1.$$

- $$x-2>0,$$
$$x>2.$$

Therefore the domain of f is $(2,3)\cup(3,\infty)$.

3. Find the slope of the line tangent to the curve $f(x)=\sqrt{x}$ at the point $(9,3)$ by using the limit definition of the instantaneous rate of change. Then find the equation of this tangent line.

We will calculate the limit $\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ to get the general formula for the slope of the tangent line for $f(x)=\sqrt{x}$. Then, we will plug in $x=9$ to this formula to get the slope of the tangent line at the point $(9, 3)$.

$$\begin{aligned}
\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}, \\
&= \lim_{h\to 0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}, \\
&= \lim_{h\to 0}\frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}, \\
&= \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}, \\
&= \frac{1}{2\sqrt{x}}.
\end{aligned}$$

Therefore, the slop of the tangent line at the point $(9, 3)$ is $\frac{1}{2\sqrt{9}} = \frac{1}{6}$.

Next, since this tangent line has slope $\frac{1}{6}$, and it passes the point $(9, 3)$, by the point-slope formula, the tangent line equation is $y - 3 = \frac{1}{6}(x - 9)$. That is, $y = \frac{1}{6}x + \frac{3}{2}$