Name :

MATH 235 Calculus 1
Quiz 3
09/27/2010

Show all work to receive full credit. Carefully write down your thought process. Your solution must not contain any logical errors. Good luck!

1. By using the precise definition of limits, prove that $\lim_{x\to 1} f(x) = 1$ if

$$f(x) = \begin{cases} x^2 & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$$

Proof. Given $\epsilon > 0$, pick $\delta > 0$ to be the smaller number between 1 and $\frac{\epsilon}{3}$. Then δ satisfies $\delta \leq 1$ and $\delta \leq \frac{\epsilon}{3}$, so if $0 < |x-1| < \delta$ then

$$\begin{aligned} |f(x) - 1| &= |x^2 - 1| \\ &= |x-1||x+1| \\ &\leq |x-1| \cdot 3 && \text{since } \delta \leq 1, \\ &< \delta \cdot 3 \\ &\leq \frac{\epsilon}{3} \cdot 3 && \text{since } \delta \leq \frac{\epsilon}{3} \\ &= \epsilon \end{aligned}$$

Note that the first inequality above holds, since $|x-1| < \delta \leq 1$ so that

$$\begin{gathered} |x-1| < 1 \\ -1 < x - 1 < 1 \\ 1 < x + 1 < 3 \end{gathered}$$

This completes the proof. □

2. Find $\lim_{x\to-\infty} \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

$$\lim_{x\to-\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = \lim_{t\to\infty} \frac{e^{-t} - e^t}{e^{-t} + e^t} = \lim_{t\to\infty} \frac{e^t(e^{-2t} - 1)}{e^t(e^{-2t} + 1)} = \lim_{t\to\infty} \frac{e^{-2t} - 1}{e^{-2t} + 1} = \frac{0-1}{0+1} = -1$$

3. Find $\dfrac{dy}{dt}$ if $y = e^{\cos^2(\pi t - 1)}$.

$$\begin{aligned} \frac{dy}{dt} &= e^{\cos^2(\pi t-1)} \frac{d}{dt} \cos^2(\pi t - 1) \\ &= e^{\cos^2(\pi t-1)} 2\cos(\pi t - 1) \frac{d}{dt} \cos(\pi t - 1) \\ &= e^{\cos^2(\pi t-1)} 2\cos(\pi t - 1)(-\sin(\pi t - 1)) \frac{d}{dt}(\pi t - 1) \\ &= -2\pi e^{\cos^2(\pi t-1)} \cos(\pi t - 1)\sin(\pi t - 1) \end{aligned}$$