MATH 235 Calculus 1
Quiz 4
10/04/2010

Show all work to receive full credit. Carefully write down your thought process. Your solution must not contain any logical errors. Good luck!

1. $y = (\frac{4t}{t+1})^{-2}$. Find $\frac{dy}{dt}$.

$$\begin{aligned}
\frac{dy}{dt} &= -2(\frac{4t}{t+1})^{-3}\frac{d}{dt}(\frac{4t}{t+1}) && \text{by the Chain Rule,}\\
&= -2(\frac{4t}{t+1})^{-3}\frac{4(t+1)-4t\cdot 1}{(t+1)^2} && \text{by the Quotient Rule,}\\
&= -2(\frac{t+1}{4t})^{3}\frac{4}{(t+1)^2}\\
&= \frac{-8(t+1)}{(4t)^3}\\
&= \frac{-t-1}{8t^3}
\end{aligned}$$

2. $y = x^2\sin^2(2x^2)$. Find $\frac{dy}{dx}$.

$$\begin{aligned}
\frac{dy}{dx} &= 2x\sin^2(2x^2) + x^2\frac{d}{dx}(\sin(2x^2))^2 && \text{by the Product Rule,}\\
&= 2x\sin^2(2x^2) + x^2(2\sin(2x^2)\frac{d}{dx}(\sin(2x^2))) && \text{by the Chain Rule,}\\
&= 2x\sin^2(2x^2) + 2x^2\sin(2x^2)\cos(2x^2)4x && \text{by the Chain Rule,}\\
&= 2x\sin(2x^2)(\sin(2x^2) + 4x^2\cos(2x^2))
\end{aligned}$$

3. Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x-axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

Since all points on the x-axis have zero y-value, we let $y = 0$ in $x^2 + xy + y^2 = 7$ and solve:

$$\begin{aligned}
x^2 &= 7\\
x &= \pm\sqrt{7}.
\end{aligned}$$

Therefore, $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ are the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x-axis. Next, we find $\frac{dy}{dx}$ and show that $\frac{dy}{dx}|_{(\sqrt{7},0)} = \frac{dy}{dx}|_{(-\sqrt{7},0)}$.

By taking the derivative of both the left hand side and the right hand side of $x^2 + xy + y^2 = 7$ with respect to x, we get

$$\begin{aligned} 2x + (y + x\frac{dy}{dx}) + 2y\frac{dy}{dx} &= 0 \\ x\frac{dy}{dx} + 2y\frac{dy}{dx} &= -2x - y \\ (x + 2y)\frac{dy}{dx} &= -2x - y \\ \frac{dy}{dx} &= \frac{-2x - y}{x + 2y} \end{aligned}$$

Therefore,

$$\frac{dy}{dx}|_{(\sqrt{7},0)} = -2 = \frac{dy}{dx}|_{(-\sqrt{7},0)},$$

which shows that the tangents to the curve at these two points are parallel, since the tangent lines at these points have a common slope.