MATH 235 Calculus 1
Quiz 5

1. At time t, the position of a body moving along the s-axis is $s = t^3 - 6t^2 + 9t$. Time is given in seconds and distance is given in meters.

a. Find the body's acceleration each time the velocity is zero.

Since

$$v(t) = s'(t) = 3t^2 - 12t + 9 \text{ and } a(t) = s''(t) = 6t - 12,$$

$v(t) = 3(t^2 - 4t + 3) = 3(t-1)(t-3) = 0$ when $t = 1$ or $t = 3$. Thus, the answer is $a(1) = -6m/sec$ and $a(3) = 6m/sec$.

b. Find the body's speed each time the acceleration is zero.

Since $a(t) = 6t - 12 = 0$ when $t = 2$ and the speed function is $sp(t) = |v(t)|$, the answer is $|v(2)| = 3m/sec$.

2. $x = 2t^2 + 3$ and $y = t^4$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $t = -1$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^3}{4t} = t^2$$

Therefore, $\frac{dy}{dx}|_{t=-1} = 1$.

Next,

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx}) = \frac{\frac{d}{dt}(\frac{dy}{dx})}{\frac{dx}{dt}} = \frac{2t}{4t} = \frac{1}{2}.$$

Therefore, $\frac{d^2y}{dx^2}|_{t=-1} = \frac{1}{2}$.

3. Find the equation of the line normal to the curve $x^2y^2 = 9$ at the point $(-1, 3)$.
By implicit differentiation and the Product rule,

$$\begin{aligned} 2xy^2 + x^2 2y\frac{dy}{dx} &= 0 \\ x^2 2y\frac{dy}{dx} &= -2xy^2 \\ \frac{dy}{dx} &= \frac{-2xy^2}{x^2 2y}. \end{aligned}$$

Therefore, the slope of the line tangent to the curve at $(-1, 3)$ is $\frac{dy}{dx}|_{(-1,3)} = 3$. We are, however, looking for the equation of the line normal to the curve at this point, so the slope of this line will be $-\frac{1}{3}$, and the normal line equation is

$$y - 3 = -\frac{1}{3}(x + 1).$$