MATH 235 Calculus 1
Quiz 6 Solution

1. Find the following values.

a. $\sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$.

b. $\cos^{-1}(-\frac{1}{\sqrt{2}}) = \frac{3\pi}{4}$

Note that for both part a and part b, there is one and only one answer, since if you "plug in" any x-value into $\sin^{-1}$ the function value will always be in $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and if you "plug in" any x-value into $\cos^{-1}$ the function value will always be in $[0, \pi]$.

2. Find the derivative of y with respect to x.

a. $y = \ln(\tan^{-1} x)$

$$\frac{dy}{dx} = \frac{1}{\tan^{-1} x} \cdot \frac{1}{1+x^2}.$$

b. $y = 6 \sinh \frac{x}{3}$

$$\frac{dy}{dx} = 6 \cosh \frac{x}{3} \cdot \frac{1}{3} = 2 \cosh \frac{x}{3}$$

3. Solve the following problems.

a. Simplify $(\sinh x + \cosh x)^4$.

$$\begin{aligned}(\sinh x + \cosh x)^4 &= (\frac{e^x - e^{-x}}{2} + \frac{e^x + e^{-x}}{2})^4 \\ &= (\frac{e^x - e^{-x} + e^x + e^{-x}}{2})^4 \\ &= (\frac{2e^x}{2})^4 \\ &= (e^x)^4 = e^{4x}.\end{aligned}$$

b. $\lim_{x \to 1^-} \sin^{-1} x = \frac{\pi}{2}$.

You can answer this question by looking at the graph $y = \sin x$, but since we have to find the limit of the inverse of $\sin x$, we have to see what x-value we are approaching as the y-value of $\sin x$ approaches 1 from the left. Your answer is, therefore, $\frac{\pi}{2}$. Note that you can only approach 1 from the left, since we restrict the graph $y = \sin x$ to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ in order to define $y = \sin^{-1} x$.