MATH235 Calculus 1
Quiz 7
11/01/2010

Show all work to receive full credit. Carefully write down your thought process. Your solution must not contain any logical errors. Good luck!

1. Estimate the volume of material in a cylindrical shell with height 30in., radius 6in., and shell thickness 0.5in.

We estimate the volume of material by using differentials dV with $r = 6$ and $dr = 0.5$. The volume of a cylindrical shell is $V = \pi r^2 h$, so $\dfrac{dV}{dr} = 2\pi r h$. Therefore,

$$\begin{aligned} dV &= 2\pi r h dr \\ &= 2\pi \cdot 6 \cdot 30 \cdot 0.5 \\ &= 180\pi. \end{aligned}$$

2. Find the absolute maximum and minimum valeus of $g(x) = e^{-x^2}$ on the closed interval $[-2, 1]$.

Since g is continuous on the closed interval $[-2, 1]$, by the extreme value theorem, we know that the absolute maximum and the absolute minimum must exist. These absolute extreme values occur either at the end points or at the critical numbers. Since

$$g'(x) = -2xe^{-x^2},$$

$g'(x) = 0$ when $x = 0 \in [-2, 1]$ and there is no x-value where $g'(x)$ does not exist. Therefore, $x = 0$ is the only critical number. Thus, we compare the function values at $x = 0$, $x = -2$, and $x = 1$:

$$\begin{aligned} g(0) &= e^0 = 1 \\ g(1) &= e^{-1} = \frac{1}{e} \\ g(-2) &= e^{-4} = \frac{1}{e^4}. \end{aligned}$$

Hence, the absolute maximum value is 1 and it occurs at $x = 0$, and the absolute minimum value is $\dfrac{1}{e^4}$ and it occurs at $x = -2$.

3. Find the value or values of c that satisfies the equation

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

in the conclusion of the Mean Value Theorem for the function $f(x) = \sin^{-1} x$ on $[-1, 1]$.

For this problem, we have to find at least one $c \in (-1, 1)$ such that

$$f'(c) = \frac{\sin^{-1} 1 - \sin^{-1}(-1)}{1 - (-1)} = \frac{\frac{\pi}{2} - (-\frac{\pi}{2})}{2} = \frac{\pi}{2}.$$

Since $f'(x) = \dfrac{1}{\sqrt{1-x^2}}$, this means that we have to find some c such that

$$\frac{1}{\sqrt{1-c^2}} = \frac{\pi}{2}.$$

Therefore,

$$\begin{aligned} \frac{1}{1-c^2} &= \frac{\pi^2}{4} \\ 1-c^2 &= \frac{4}{\pi^2} \\ 1-\frac{4}{\pi^2} &= c^2 \\ c &= \pm\sqrt{1-\frac{4}{\pi^2}}. \end{aligned}$$