MATH235 Calculus 1
Quiz 8
11/10/2010

Show all work to receive full credit. Carefully write down your thought process. Your solution must not contain any logical errors. Good luck!

1. Find the absolute extreme values of $f(x) = \ln(\cos x)$ on $[-\frac{\pi}{4}, \frac{\pi}{3}]$.

$f(x) = \ln(\cos x)$ is continuous on the closed interval $[-\frac{\pi}{4}, \frac{\pi}{3}]$, so by the extreme value theorem, we know that the absolute maximum value and the absolute minimum value of this function on $[-\frac{\pi}{4}, \frac{\pi}{3}]$ do exist. The only candidates where this function can have such extreme values are the critical numbers or the end points of the given closed interval. Since $f'(x) = -\frac{\sin x}{\cos x}$, $f'(x) = 0$ when $x = 0 \in [-\frac{\pi}{4}, \frac{\pi}{3}]$. We also check when $f'(x)$ does not exist on $[-\frac{\pi}{4}, \frac{\pi}{3}]$, but this will never happen on this interval. Therefore, the only critical number is $x = 0$. Hence, we compare

$$f(0) = \ln 1 = 0$$
$$f(-\frac{\pi}{4}) = \ln \frac{1}{\sqrt{2}}$$
$$f(\frac{\pi}{3}) = \ln \frac{1}{2}.$$

Since $\frac{1}{2} < \frac{1}{\sqrt{2}} < 1$, and $y = \ln x$ is an increasing function, we have that $\ln \frac{1}{2} < \ln \frac{1}{\sqrt{2}} < \ln 1$. Therefore, the absolute maximum value of $f(x) = \ln(\cos x)$ on $[-\frac{\pi}{4}, \frac{\pi}{3}]$ is $\ln 1 = 0$, and the absolute minimum value is $\ln \frac{1}{2}$.

2. Find all local extreme values of $f(x) = xe^{\frac{1}{x}}$.

All local extreme values occur at critical numbers (Not all critical numbers give local extreme values though!), so we first find all critical numbers. Then we will check at which of these critical numbers f has a local extreme value. First of all, note that the domain of f is $(-\infty, 0) \cup (0, \infty)$. Since

$$f'(x) = e^{\frac{1}{x}} + xe^{\frac{1}{x}}(-\frac{1}{x^2}) = e^{\frac{1}{x}}(1 - \frac{1}{x}),$$

$f'(x) = 0$ when $x = 1$ and $f'(x)$ does not exist when $x = 0$, but critical numbers must be in the domain of f, so $x = 1$ is the only critical number. The sign of f' changes from negative to positive at $x = 1$, so by the first derivative test for local extrema, f has a local minimum at $x = 1$, and the local minimum values is $f(1) = e$.

3. When is the parabola $y = ax^2 + bx + c, a \neq 0$ concave up? When is it concave down? Explain your answer.

If $y = f(x)$ is twice differentiable, then the graph of f is concave up when $f''(x) > 0$ and concave down when $f''(x) < 0$. Let $f(x) = ax^2 + bx + c$. Then

$$f'(x) = 2ax + b$$
$$f''(x) = 2a,$$

so the graph of f is concave up when $2a > 0$, i.e., when $a > 0$, and it is concave down when $2a < 0$, i.e., $a < 0$.