Integration Review

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  * Integration by Parts
  * Trigonometric Integration
  * Trigonometric Substitution
  * Partial Fractions.

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- Simplify the integrand if possible.
- Look for an obvious or a clever substitution.
- Decide how you are going to integrate:
  * Integration by Parts
  * Trigonometric Integration
  * Trigonometric Substitution
  * Partial Fractions.
- If none of the above work, try to manipulate the integrand to transform it into an easier form which you can apply the above techniques.

Integration Review : Simplify the Integrand if Possible.

- $\int (x + \sin x)^2 dx$

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- $\int (x+\sin x)^2 dx$
- $= \int x^2 + 2x\sin x + \sin^2 x dx$
- 1. $\int x^2 dx = \frac{1}{3}x^3 + C$

  2. $\int 2x\sin x dx = 2\int x\sin x dx$

  $= 2(-x\cos x - \int(-\cos x)\cdot 1 dx)$ by Integration by Parts,

  $= -2x\cos x + 2\sin x + C$

  3. $\int \sin^2 x dx$

  $= \int \frac{1-\cos(2x)}{2} dx = \frac{1}{2}(x - \frac{1}{2}\sin(2x)) + C$

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- 1. $\int x^2 dx = \dfrac{1}{3}x^3 + C$

  2. $\int 2x \sin x dx = 2 \int x \sin x dx$

  $= 2(-x \cos x - \int (-\cos x) \cdot 1 dx)$ by Integration by Parts,

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  3. $\int \sin^2 x dx$

  $= \int \dfrac{1 - \cos(2x)}{2} dx = \dfrac{1}{2}(x - \dfrac{1}{2} \sin(2x)) + C$
- Therefore,

  $\int (x + \sin x)^2 dx = \dfrac{1}{3}x^3 - 2x \cos x + 2 \sin x + \dfrac{1}{2}x - \dfrac{1}{4} \sin(2x) + C$

Integration Review : Simplify the integrand if Possible.

Try $\int \frac{\sec x \cos(2x)}{2\sin x + \sec x} dx$

Integration Review: u-Substitution.

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Integration Review: u-Substitution.

- $\int \frac{1}{\sqrt{x} + x\sqrt{x}} dx$
- Let $u = \sqrt{x}$ so that $du = \frac{1}{2\sqrt{x}} dx$ and $x = u^2$.

Integration Review: u-Substitution.

- $\int \frac{1}{\sqrt{x} + x\sqrt{x}} dx$
- Let $u = \sqrt{x}$ so that $du = \frac{1}{2\sqrt{x}} dx$ and $x = u^2$.
- Then
$$\int \frac{1}{\sqrt{x} + x\sqrt{x}} dx = \int \frac{1}{(1+x)\sqrt{x}} dx$$

Integration Review: u-Substitution.

- $\int \frac{1}{\sqrt{x}+x\sqrt{x}}dx$
- Let $u = \sqrt{x}$ so that $du = \frac{1}{2\sqrt{x}}dx$ and $x = u^2$.
- Then
  $\int \frac{1}{\sqrt{x}+x\sqrt{x}}dx = \int \frac{1}{(1+x)\sqrt{x}}dx$
- $= \int \frac{1}{1+u^2}2du = 2\int \frac{1}{1+u^2}du$

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- $\int \frac{1}{\sqrt{x} + x\sqrt{x}} dx$
- Let $u = \sqrt{x}$ so that $du = \frac{1}{2\sqrt{x}} dx$ and $x = u^2$.
- Then $\int \frac{1}{\sqrt{x} + x\sqrt{x}} dx = \int \frac{1}{(1+x)\sqrt{x}} dx$
- $= \int \frac{1}{1+u^2} 2du = 2 \int \frac{1}{1+u^2} du$
- $= 2 \tan^{-1} u + C = 2 \tan^{-1} \sqrt{x} + C$

Integration Review: u-Substitution.

Try $\int \tan^{-1} \sqrt{x} dx$

Try $\int e^{\sqrt{x}} dx$

Integration Review: u-Subsitution and Trigonometric Substitution.

- $\int \frac{1}{x^2\sqrt{4x^2+1}}dx$

Integration Review: u-Subsitution and Trigonometric Substitution.

- $\displaystyle\int \frac{1}{x^2\sqrt{4x^2+1}}dx$
- Try with $u = \sqrt{4x^2+1}$ and complete the integration with a trigonometric substitution.

Clever u-Substitution

- $\int \frac{\ln(\tan x)}{\sin x \cos x} dx$

Clever u-Substitution

- $\int \frac{\ln(\tan x)}{\sin x \cos x} dx$
- Let $u = \ln(\tan x)$ so that
  $du = \frac{1}{\tan x} \cdot \sec^2 x dx = \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} dx = \frac{1}{\sin x \cos x} dx$.

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  $du = \frac{1}{\tan x} \cdot \sec^2 x dx = \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} dx = \frac{1}{\sin x \cos x} dx$.
- Then
  $\int \frac{\ln(\tan x)}{\sin x \cos x} dx = \int u du$
- $= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln(\tan x))^2 + C$.

Clever u-Substitution.

Try $\int \cos x \cos^3(\sin x)dx$

Try $\int \frac{1}{1+2e^x - e^{-x}}dx$

Integration by Parts

- $\int \frac{\ln(x+1)}{x^2} dx$

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- $= -\frac{1}{x}\ln(x+1) - \int (-\frac{1}{x})\frac{1}{x+1} dx$ by Integration by Parts,

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- $= -\frac{1}{x}\ln(x+1) + \int \frac{1}{x(x+1)}dx$

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- $= -\frac{1}{x}\ln(x+1) + \ln|x| - \ln|x+1| + C$

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- $= -\frac{1}{x}\ln(x+1) + \int \frac{1}{x} - \frac{1}{x+1} dx$ by Partial Fractions,
- $= -\frac{1}{x}\ln(x+1) + \ln|x| - \ln|x+1| + C$
- $= \ln \frac{x}{(x+1)^{\frac{1}{x}}(x+1)} + C$

Trigonometric Integration

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- $= \int (\tan^2 x + 1)\sec^2 x - 2\sec^2 x + 1 dx$

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- $= \int (\sec^2 x - 1)^2 dx$
- $= \int \sec^4 x - 2\sec^2 x + 1 dx$
- $= \int (\tan^2 x + 1)\sec^2 x - 2\sec^2 x + 1 dx$
- $= \dfrac{1}{3}\tan^3 x + \tan x - 2\tan x + x + C$

Trigonometric Integration

- $\int \tan^4 x dx$
- $= \int (\tan^2 x)^2 dx$
- $= \int (\sec^2 x - 1)^2 dx$
- $= \int \sec^4 x - 2\sec^2 x + 1 dx$
- $= \int (\tan^2 x + 1)\sec^2 x - 2\sec^2 x + 1 dx$
- $= \frac{1}{3}\tan^3 x + \tan x - 2\tan x + x + C$
- $= \frac{1}{3}\tan^3 x - \tan x + x + C$.

Trigonometric Integration

Try $\int_0^{\frac{\pi}{4}} \tan^5\theta \sec^3\theta d\theta$

Partial Fractions

Try $\int \frac{1}{x(x^4+1)}dx$