Group work problems. (3/24)

* Determine whether the series converges or diverges. Give reason to your answer

① $\sum_{n=1}^{\infty} \frac{2^n}{n+1}$: $\boxed{\text{DIVERGENT}}$.

since $\lim_{n\to\infty} \frac{2^n}{n+1} = \lim_{\substack{x\to\infty \\ x\in\mathbb{R}}} \frac{2^x}{x+1} \underset{L.R.}{=} \lim_{x\to\infty} \frac{2^x \ln 2}{1} = \infty \neq 0$.

② $\sum_{n=1}^{\infty} \frac{1}{n(1+(\ln n)^2)}$

i) $a_n = \frac{1}{n(1+(\ln n)^2)}$ is decreasing. ✓

ii) // is positive. ✓

$\}$ So we can apply the Integral Test.

$$\int_1^{\infty} \frac{1}{x(1+(\ln x)^2)}\,dx = \lim_{t\to\infty} \int_1^{t} \frac{1}{x(1+(\ln x)^2)}\,dx$$

$u = \ln x$
$du = \frac{1}{x}dx$
$\int_1^t \to \int_0^{\ln t}$

$$= \lim_{t\to\infty} \int_0^{\ln t} \frac{1}{1+u^2}\,du$$

$$= \lim_{t\to\infty} \left[\tan^{-1} u\right]_0^{\ln t}$$

$$= \lim_{t\to\infty} \left(\tan^{-1}(\ln t) - \underbrace{\tan^{-1} 0}_{=0}\right)$$

$$= \lim_{t\to\infty} \tan^{-1}(\ln t) = \tan^{-1}\Big(\underbrace{\lim_{t\to\infty} \ln t}_{\infty}\Big) = \frac{\pi}{2}$$

Since $\int_1^{\infty} \frac{1}{x(1+(\ln x)^2)}\,dx$ is convergent,
by the Integral Test, $\sum_{n=1}^{\infty} \frac{1}{n(1+(\ln n)^2)}$ is also $\boxed{\text{CONVERGENT}}$.

③ $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$

$0 \le \frac{1}{n(n+1)(n+2)} \le \frac{1}{n^3}$

$\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges since $\int_1^{\infty} \frac{1}{x^3}\,dx$ converges.
(Integral Test)

$\}$ Thus, $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$ also converges by the Comparison Test.