Worksheet 2 MATH 235 Name: ____________________

Fall, 2010.

Discuss the following problems with your group and write down a complete solution. Show all work.

1. Prove the triangle-inequality: For all $x, y \in \mathbb{R}$

$$|x+y| \leq |x| + |y|.$$

- step 1. Discuss with your group if $-|x| \leq x \leq |x|$ is always true.
- step 2. Write down a similar inequality for y as well.
- step 3. Add the two inequalities from step 1 and 2.
- What can you conclude from this final inequality?

2. Suppose $f(x) \leq g(x)$ for all $x \in \mathbb{R}$ and that $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$. Show that $L \leq M$.

- step 1. Suppose $L > M$. Our goal is to reach a contradiction with this assumption to conclude that $L \leq M$.
- step 2. Recall that $\lim_{x \to c}(g(x) - f(x)) = M - L$ and write down the precise definition of $\lim_{x \to c}(g(x) - f(x)) = M - L$ using ϵ and δ.
- step 3. Pick $\epsilon = L - M$ in step 2, which is possible since we are assuming that $L - M > 0$. Then, by using using the fact that $a \leq |a|$ for all $a \in \mathbb{R}$, conclude that $(g(x) - f(x)) - (M - L) < L - M$ whenever $0 < |x - c| < \delta$.
- step 4. Explain why this is a contradiction and conclude your argument.

Observe from your proof that this statement holds true when $f(x) \leq g(x)$ for all x in an open interval containing c (except possibley at c itself) although we have assumed that $f(x) \leq g(x)$ for all $x \in \mathbb{R}$.