Worksheet 4 MATH 235 Name: _____

Fall, 2010.

Discuss the following problems with your group and write down a complete solution. Show all work.

1. Prove that if f is differentiable at $x = c$, then f is continuous at $x = c$.
   - step 1. How do you show that f is continuous at $x = c$? Write down what you have to prove.
   - step 2. Rewrite the limit appearing in step 1 by using a new variable $h = x - c$.
   - step 3. Discuss with your group how the following is true: if $h \neq 0$, then
   $$\begin{aligned} f(c+h) &= f(c+h) - f(c) + f(c), \\ &= \frac{f(c+h) - f(c)}{h} \cdot h + f(c). \end{aligned}$$
   - step 4. By using step 3, find $\lim_{h\to 0} f(c+h)$. Discuss where you are using the given information that f has a derivative at $x = c$.
   - Put all of the above steps together and write down a complete mathematical proof.

2. Is the converse of the previous statement true? In other words, is it true that if f is continuous at $x = c$, then f is differentiable at $x = c$? We can show that this statement is FALSE by finding a counterexample. Show that $f(x) = |x|$ is continuous at $x = 0$ but it is not differentiable at $x = 0$.