Worksheet 6 MATH 235
10/21/2010

Discuss the following problems with your group and write down a complete solution. Show all work.

1. (The Derivative Rule for Inverses.) Let f be a continuous one-to-one function defined on an interval. Suppose f is differentiable at $x = a$ and $f'(a) \neq 0$. If $f(a) = b$, show that $(f^{-1})'(b)$ exists and that

$$(f^{-1})'(b) = \frac{1}{f'(f^{-1}(b))}.$$

- step 1. Write down the limit definition of $(f^{-1})'(b)$.

- step 2. Pick $k \in \mathbb{R}$ so that $f^{-1}(b+h) = a + k$ (equivalently $f(a+k) = b+h$) and continue from step 1 to reach:

$$(f^{-1})'(b) = \lim_{h \to 0} \frac{k}{f(a+k) - f(a)}$$

- step 3. If h approaches zero, what number does $k = f^{-1}(b+h) - a$ approach? You will have to use the fact that f^{-1} is continuous. This is true, since it is given that f is continuous. Continuing from step 2, conclude that

$$(f^{-1})'(b) = \lim_{k \to 0} \frac{k}{f(a+k) - f(a)} = \frac{1}{f'(a)} = \frac{1}{f'(f^{-1}(b))}.$$

- By putting the above steps together, write down a complete mathematical proof.

2. From the previous problem, we now know that

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

for all x in the domain of f^{-1}. By using this result, derive the derivative of the following functions.

- $y = \ln x$
- $y = \log_2 x$
- $y = \sin^{-1} x$
- $y = \cos^{-1} x$
- $y = \tan^{-1} x$