

Worksheet 7 MATH 235
10/28/2010

Discuss the following problems with your group and write down a complete solution. Show all work.

1. (Rolle's Theorem) Suppose $y = f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) . If $f(a) = f(b)$, then there exists at least one number $c \in (a, b)$ such that $f'(c) = 0$.

2. (The Mean Value Theorem) Suppose $y = f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) . Then, there exists at least one number $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

- step 1. Consider the line joining $(a, f(a))$ and $(b, f(b))$. If this line is the graph of the function $y = g(x)$, find a formula for $g(x)$.

- step 2. Define $h(x) = f(x) - g(x)$. Why is h continuous on $[a, b]$? Why is h differentiable on (a, b) ?

- step 3. Find $h(a)$ and $h(b)$.

- step 4. Notice that we can apply Rolle's Theorem to h by step 2 and step 3. What does Rolle's Theorem tell you?

- step 5. What is $h'(x)$? Conclude that there exists a c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

- By putting the above steps together, write down a complete mathematical proof.