Worksheet 8 MATH 235
11/11/2010

Discuss the following problems with your group and write down a complete solution. Show all work.

1. (The First Derivative Test for Local Extrema) Suppose $x = c$ is a critical number of a continuous function f, and f is differentiable at every point in some interval containing $x = c$ except possibly at c itself. Moving across $x = c$ from left to right,

- if f' changes from negative to positive at $x = c$, then f has a local minimum at $x = c$.
- if f' changes from positive to negative at $x = c$, then f has a local maximum at $x = c$.
- if f' does not change signs at $x = c$, then f has no local extremum at $x = c$.

2. (The Second Derivative Test for Local Extrema) Suppose f'' is continuous on an open interval that contains $x = c$.

- If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.
- If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.
- If $f'(c) = 0$ and $f''(c) = 0$, then f may have a local maximum, a local minimum, or neither at $x = c$.

Prove the Second Derivative Test for Local Extrema.

- step 1. If $f''(c) < 0$, is it true that there exists an interval J containing $x = c$ such that $f''(x) < 0$ for all x in this interval? Why?

- step 2. What does this tell you about f' on J? Complete the proof of the first assertion.

- step 3. By the same way as in step 1 and step 2, complete the proof of assertion 2.

- step 4. Prove assertion 3.