Is Seeing Believing in Geometry?

Byrne's Euclid in Color

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Axiomatic Framework for Bob the Young Bus Delinquent

Axiom: Person is trustworthy unless proven otherwise.

Proposition 1: My daughter, her friend Alice and Alice's sister are trustworthy

Proof: We have no reason to think otherwise and so, according to the Axiom, they must be so.

Proposition 2: Bob the Delinquent is in fifth grade.

Proof: By Proposition 1, Alice's sister knows and is truthful about who is in the same class as her. Alice knows and is truthful about what grade her sister is in. Bob is in the same class as Alice's sister and so must be in the fifth grade.

The Euclidean Model of Reason

Euclid's *Elements* Axiomatic treatment of geometry of lines, circles, triangles, and more.

Pythagorean Theorem, π is a constant, and much more.

Trust but Verify Do the rails of train tracks meet?
How to tell if two fields have the same area?

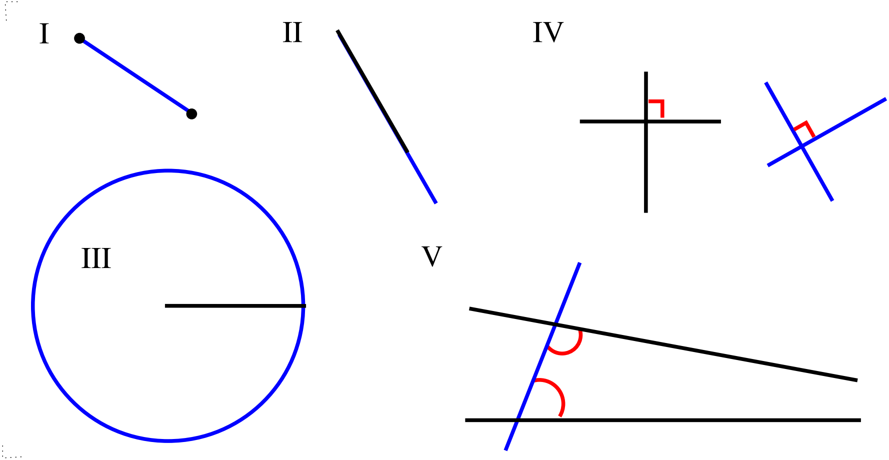
Plato *Let none ignorant of geometry enter.*

Spinoza aims to understand human emotions *as if the surfaces of lines, planes or solids.*

Kant *absolute universality which are characteristic of all propositions of geometry.*

Euclid pivotal to Ancient and Classical education.

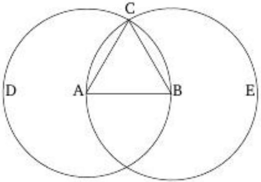
The Five Postulates



An Equilateral Triangle

Proposition 1

To construct an equilateral triangle on a given finite straight-line.



Let AB be the given finite straight-line.

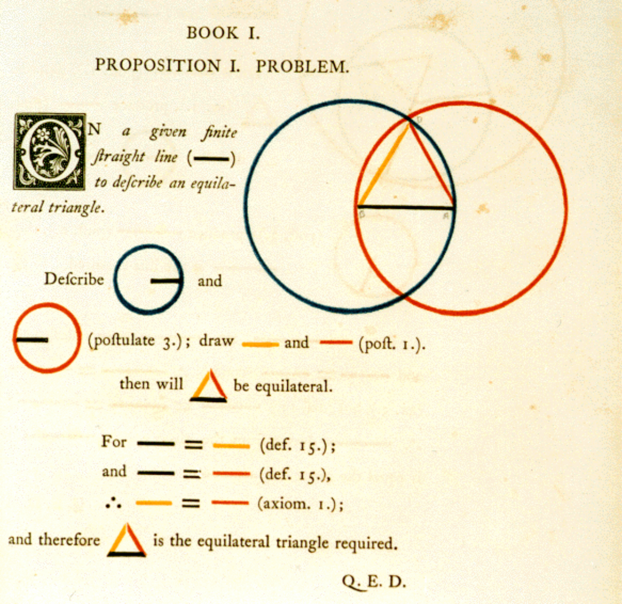
So it is required to construct an equilateral triangle on the straight-line AB.

Let the circle BCD with center A and radius AB have been drawn [Post. 3], and again let the circle ACE with center B and radius BA have been drawn [Post. 3]. And let the straight-lines CA and CB have been joined from the point C, where the circles cut one another,[†] to the points A and B (respectively) [Post. 1].

And since the point A is the center of the circle CDB, AC is equal to AB [Def. 1.15]. Again, since the point B is the center of the circle CAE, BC is equal to BA [Def. 1.15]. But CA was also shown (to be) equal to AB. Thus, CA and CB are each equal to AB. But things equal to the same thing are also equal to one another [C.N. 1]. Thus, CA is also equal to CB. Thus, the three (straight-lines) CA, AB, and BC are equal to one another.

Thus, the triangle ABC is equilateral, and has been constructed on the given finite straight-line AB. (Which is) the very thing it was required to do.

An Equilateral Triangle



BOOK I.

PROPOSITION I. PROBLEM.

ON *a given finite ſtraight line* ( ) *to deſcribe an equilateral triangle.*

Deſcribe and (poſtulate 3.); draw and (poſt. 1.).

then will be equilateral.

For = (def. 15.);

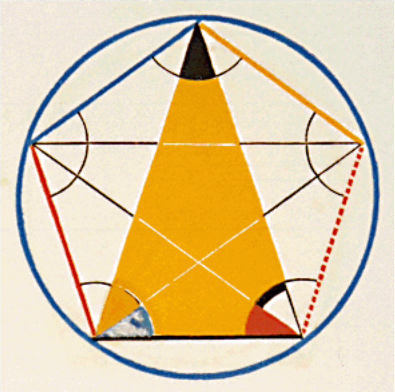
and = (def. 15.),

∴ = (axiom. 1.);

and therefore is the equilateral triangle required.

Q. E. D.

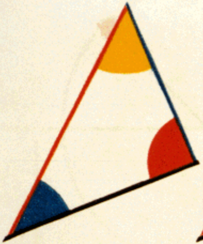
Byrne's Euclid (1847)



- *Most works addressed to the proficient, not the uninitiated.*
- *... as I should have explained it orally with my pupils.*
- *Dyed chalks and coloured pencils will be very convenient.*

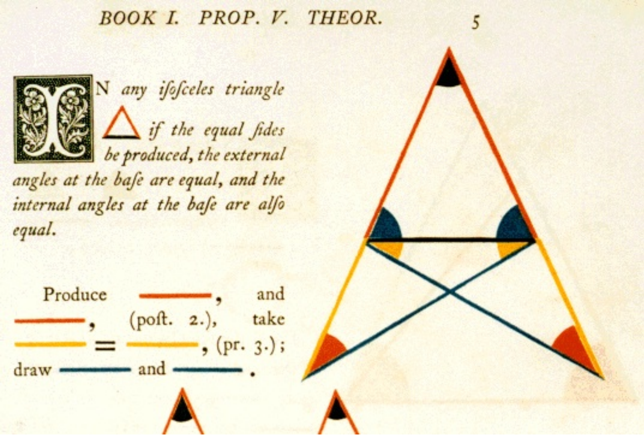
SAS Congruence

4 BOOK I. PROP. IV. THEOR.



IF two triangles have two ſides of the one reſpectively equal to two ſides of the other, (to and to) and the angles (and) contained by thoſe equal ſides alſo equal; then their baſes or their ſides (and) are alſo equal: and the remaining and their remaining angles oppoſite to equal ſides are reſpectively equal (= and =): and the triangles are equal in every reſpect.

Isosceles Triangles



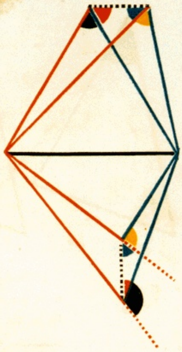
BOOK I. PROP. V. THEOR. 5

IN *any isosceles triangle* if the equal sides be produced, the external angles at the base are equal, and the internal angles at the base are also equal.

Produce , and , (post. 2.), take = , (pr. 3.); draw and .

SSS Congruence

ON the ſame baſe (), and on the ſame ſide of it there cannot be two triangles having their conterminous ſides (and , and) at both extremities of the baſe, equal to each other.



When two triangles ſtand on the ſame baſe, and on the ſame ſide of it, the vertex of the one ſhall either fall outſide of the other triangle, or within it; or, laſtly, on one of its ſides.

If it be poſſible let the two triangles be conſtructed ſo that { = , = }, then draw and,

= (pr. 5.)

∴ ⊐ and

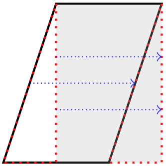
∴ ⊐ } which is abſurd,

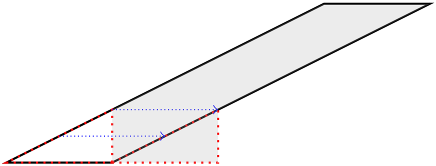
but (pr. 5.) =

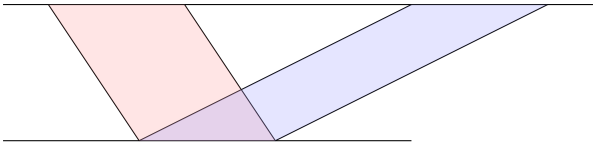
therefore the two triangles cannot have their conterminous ſides equal at both extremities of the baſe.

Q. E. D.

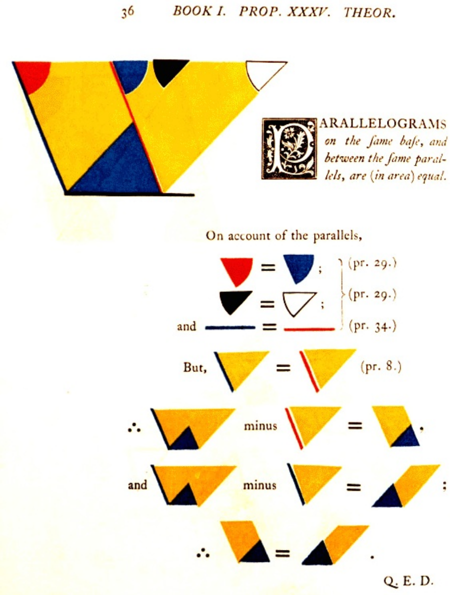
Areas of Parallelograms







Areas of Parallelograms



PARALLELOGRAMS on the ſame baſe, and between the ſame parallels, are (in area) equal.

On account of the parallels,

= ; (pr. 29.)

= ; (pr. 29.)

and = (pr. 34.)

But, = (pr. 8.)

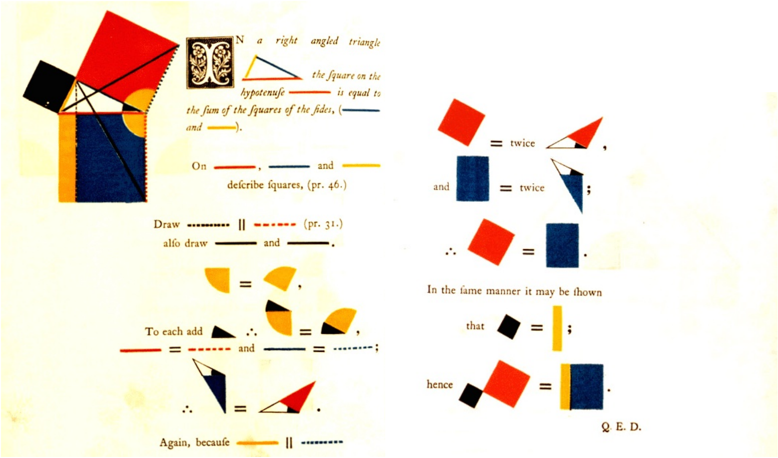
∴ minus = .

and minus = ;

∴ = .

Q. E. D.

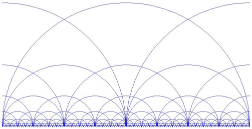
Pythagorean Theorem



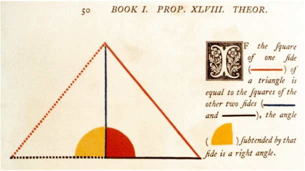
Visual comprehension ⟷ Geometric Rigor

The Decline of Euclid in the Victorian Age

1850's Euclid as best seller (c.f. G. Eliot's *Daniel Deronda*)



- Philosophical The emergence of Non-Euclidean Geometry. The Kantian exhortation of the universality of geometry no longer held.



- Societal Liberal $\longrightarrow$ Vocational. Practical aspects of Euclid were on the rise. It was assumed that artisans were only interested in direct applications.

Byrne's Euclid: The Wrong Place at the Wrong Time

- Between a technological rock and a traditional hard place.
- Augustus de Morgan Byrne the Eccentric.
- Money Cost was three times more than the next most expensive Elements. Chiswick Press's demise.

Some (not all) references

- Alice Jenkins in *What the Victorians Learned...* Journal of Victorian Studies 2007.
- Euclid Site `http://www.math.ubc.ca/ cass/Euclid/`
- Byrne's Euclid, facsimile by Taschen.

Modern Euclidean Pleasure: Euclid Constructions Game