Mathematical Association of America Mathfest 2017, Chicago, Illinois.

Themed Contributed Paper Session - Euclid and the Mathematics of Antiquity in the 21st Century.

Time and Location: Saturday, July 29, 1:00pm – 4:20pm. Salon A-1.

Organizers: Elizabeth Theta Brown (brownet@jmu.edu) and Edwin O'Shea (osheaem@jmu.edu), James Madison University.

Session Abstract: Euclid's Elements is a fundamental text of mathematics in the western tradition. Geometry, number theory, logic, and the axiomatic method: all bear Euclids stamp. Moreover, the Elements was considered a central text of every liberal arts education well into the nineteenth century, more than two millennia after its writing. The recent centennial of the MAA provides a fitting occasion on which to revisit the influence of mathematics's past on future mathematics and culture. We seek contributions that relate the work of Euclid or other mathematicians of antiquity to modern mathematics or the modern undergraduate curriculum. Original research, unique expositions, descriptions of courses with a significant integration of the mathematics of antiquity, and curricular materials are all welcome.

1:00 PM	Andrew Leahy	Bring back the Pappus-Guldin Theorems
1:20 PM	Jerry Lodder	A Course in Geometry based on Historical Sources
1:40 PM	Viktor Blåsjö	Euclid's geometry is physical, not abstract
2:00 PM	Charlie Smith	My Big Fat Greek Course
2:20 PM	Jeffrey Clark	Rationals, Irrationals, and Commensurable Magnitudes: Euclid and the Real Numbers
2:40 PM	J Christopher Tweddle	Solving Quadratic Equations with Geometric Algebra
3:00 PM	Ian Pierce, Kurt Herzinger, Courtney Kunselman	Climbing Greek Ladders to Reach for Eigenvectors
3:20 PM	Kathi Crow	FYS: Math of the Middle East and North Africa
3:40 PM	Marshall A. Whittlesey	The mathematics of the Sphaerica of Menelaus
4:00 PM	Maureen Carroll, Elyn Rykken	Geometry: It's Element-ary

- **Bring back the Pappus-Guldin Theorems**,
  Andrew Leahy, Knox College.
  The most powerful geometrical proving technique of antiquity was the theory of proportions introduced in Books V and VI of Euclid's Elements. Archimedes was undoubtedly the most successful practitioner of this theory, using it to compute areas, centroids, and volumes and surfaces of revolution–key applications of today's integral. 600 years after Archimedes, Pappus of Alexandria demonstrated a pair of theorems–today called the Pappus-Guldin theorems–which showed that given any two of these quantities the third was easily determined. These theorems

still appear in many calculus textbooks today, but often only on the periphery. In this talk, we will show how putting the Pappus-Guldin theorems up front in calculus results in an intuitive and unified presentation of volumes and surfaces of revolution that at the same time extends the formulas we teach in calculus today.

- **A Course in Geometry based on Historical Sources**,
  Jerry Lodder, New Mexico State University.
  A course in plane geometry at the undergraduate level is taught today from a system of highly polished axioms going well beyond the postulates and common notions of Euclid's Elements. We outline the contents of a historical curricular module that examines the proof of the Pythagorean Theorem from Book I of the Elements. With the ancient Greek view of area in mind, we see how the Pythagorean Theorem becomes transparent by finding parallelograms that are on the same base and contained between the same parallel lines. Of course, the Pythagorean Theorem is used heavily today for the distance formula in Euclidean space, and depends on the parallel postulate (Postulate V, Book I). Should this postulate fail, what would replace the distance formula? What would replace area measure? The former question finds resolution in Riemannian geometry and the latter in symplectic geometry, both being examples of non-Euclidean geometry. We close with a brief excerpt from a curricular module based on the work of N. Lobachevsky and F. Klein.

- **Euclid's geometry is physical, not abstract**,
  Viktor Blåsjö, Utrecht University.
  Euclid's geometry is often taken to be eminently pure and abstract theory. This point of view is congenial to modern conceptions of mathematics, and was popular historically among philosophers such as Plato. I argue, however, that view was rejected by not only Euclid himself but also by virtually all major geometers from antiquity to Leibniz. These mathematicians instead saw the very foundations of geometry as anchored in concrete figures constructed by instruments such as ruler and compass in the physical world. I use this perspective to argue that several so-called flaws in the Elements are not flaws at all. I also refute various arguments and alleged evidence that Euclid held the Platonic view. The physicalist interpretation of classical geometry goes well with hands-on trends in modern pedagogy and vindicates it against snobbish and one-sided emphasis on an abstract and purely logical conception of mathematics.

- **My Big Fat Greek Course**,
  Charlie Smith, Park University.
  MA 350 History of Mathematics is specifically designed for mathematics and mathematics education majors at Park University. It uses the achievements of the great Greeks of antiquity as a springboard to more recent developments. For example, the construction problems which originated between 500 and 400 B.C.E. provide an opportunity to introduce the concepts of algebraic, transcendental and constructible numbers, setting the stage for the decisive work of Euler, Gauss, Wantzel and Lindemann centuries later. The course structure consists of the following

units: The Pythagoreans, Classic Construction Problems, Euclid, Archimedes, Diophantus, Fibonacci, and The Cubic Controversy. Emphasis is placed on both fact and legend. A field trip to the Linda Hall Library is required; it has a wonderful collection of classic mathematics books dating back to 1482! Films and maps are utilized to add variety and detail that would otherwise be lacking.

- **Rationals, Irrationals, and Commensurable Magnitudes: Euclid and the Real Numbers**,
  Jeffrey Clark, Elon University.
  In Book V of Euclid's The Elements he discusses the theory of ratios of magnitudes as attributed to Eudoxus. In dealing with these ratios he describes their properties in comparison with commensurable ratios, i.e., rational numbers. In essence this discussion describes the positive real numbers in terms of their relationship to the rational numbers, a process that we now refer to as Dedekind cuts. This talk will summarize Euclid's exposition in comparison with how we approach the real numbers today in a typical Analysis class.

- **Solving Quadratic Equations with Geometric Algebra**,
  J Christopher Tweddle, Governors State University.
  The Pythagoreans utilized the "application of areas" to solve algebraic expressions. Book VI of Euclid's Elements includes results in proportionality that are essential to the application of geometric algebra to solve quadratic equations. In particular, propositions 28 and 29 give constructions that lead to solutions of equations of the form $x(a-x) = b^2$ and $x(x+a) = b^2$, where a, b and c are positive numbers, respectively. The propositions will be presented and translated into modern mathematical language. The construction procedure will be demonstrated by example. Finally, connections between Euclidean geometric solutions and modern techniques for solving quadratic equations will be drawn.

- **Climbing Greek Ladders to Reach for Eigenvectors**,
  Ian Pierce, US Air Force Academy; Kurt Herzinger, US Air Force Academy; Courtney Kunselman, US Air Force Academy.
  Theons ladder (also known as a Greek ladder) is an ancient method for approximating irrational roots of integers. One can also modify a Greek ladder to approximate roots of certain polynomials. In particular, one can find previous results in the literature concerned with approximating one of the roots of a quadratic polynomial subject to certain constraints on its coefficients. We will address how one can extend these ideas to the consideration of any quadratic polynomial; in addition we will show how to adjust the Greek ladder so that it will approximate either of the roots (rather than just one). We may also explain those situations in which a Greek ladder fails to converge and discuss some ongoing related work with polynomials of higher degree. Understanding why and how these Greek ladders behave as they do requires only some basic ideas from linear algebra and a basic understanding of limits.

- **FYS: Math of the Middle East and North Africa**,
  Kathi Crow, Salem State University.
  First year seminars provide an introduction to college as well as an opportunity to delve into topics outside of the typical math curriculum. In this talk I will describe a course which touches on themes from the ancient world and uses them to introduce students to the modern university. We will discuss how the library at Alexandria was a center for learning that attracted mathematicians such as Euclid, Eratosthenes, and Hypatia just as the university library is a hub for learning on campus. As we delve into islamic tilings and connections with modern theorems we will discuss citations, plagiarism, and academic honesty. The course will end with a survey of modern mathematicians from the Middle East and North Africa to show students that not only is mathematics a vibrant subject with many avenues of research but also that math research is a global endeavor with people from all over the world contributing their ideas.

- **The mathematics of the Sphaerica of Menelaus**,
  Marshall A. Whittlesey, California State University San Marcos.
  The Sphaerica of Menelaus of Alexandria is an ancient text on spherical geometry and trigonometry. Originally written in Greek, the text was translated and heavily reworked in later centuries by Arabic mathematicians. This text is not as well known as Euclids Elements partly because spherical geometry has not been a standard part of the mathematics curriculum for many decades, but also because the rise of trigonometry has rendered many of Menelaus results and techniques less important. However, the text contains a number of intriguing results not well known to modern mathematicians that illustrate the talent of the ancient writers. We discuss some of these theorems, their proofs, and their applications.

- **Geometry: It's Element-ary**,
  Maureen Carroll, University of Scranton; Elyn Rykken, Muhlenberg College.
  The Elements is the most natural entry point into the study of axiomatic geometry, and provides the perfect scaffolding for building a compelling narrative for a self-contained course, that is, one that provides, rather than requires, a good understanding of Euclidean geometry. With this in mind, we discuss a junior/senior level geometry course developed over two decades of teaching geometry and history of mathematics courses. The course begins with Heath's translation of the first four books of the Elements and places a heavy emphasis on active participation. Specifically, our students present updated versions of many of the propositions in Books I and III. Along the way, after establishing a firm foundation of neutral geometry, we consider the behavior of the postulates beyond the tried-and-true Euclidean plane. This investigation leads students to discover the insufficiency of Euclid's set of axioms, and thus, presents an organic introduction to the work of Hilbert. In addition to discussing the course, we will detail our companion book project.