Instructor: Edwin O'Shea	*Class Meetings:* MWF 12.20-1.10 in Roop 213
	Office Hours MWF 9.30-10.50* in Roop 323 (*no shows, end at 10.30)
Email: `osheaem@jmu.edu`	*Web:* `www.educ.jmu.edu/~osheaem/teaching/310.html`

Texts and materials *Number Theory* (Dover) by Andrews. In addition, there will be some handouts given out during the semester.

Course description Math 310 is an elective in Number Theory for math majors. At its heart, Number Theory is an area where one "thinks deeply about simple things," the simple things being the integers and the wondrous relationships and intoxicating deep theorems about this most simple and primitive of sets. To some extent we'll take a *Great Questions* approach to this class – please see the *Introductory Problems* worksheet that is attached which you are required to think about and make conjectures about in the first week.

There will be a lot of problems in this course, in the sense that we will redevelop rich and old theories and apply to it to surprising contexts. The course assumes completion of Math 245, in particular, a grasp of proof by induction and knowledge of the Euclidean algorithm for integers. We will closely follow the required text, more or less going at a pace of one section a day.

Some comments about the text Andrews' *Number Theory* is a lovely text and has a tendency to abstract simple claims to the most generality and is careful in its axiomatic treatment. It gives a broad and accessible view of Number Theory, perhaps with a more combinatorial feel than other texts. Two features that distinguish it from other texts are its beautiful treatment of classical integer partitions and its use of analytic (calculus) tools, especially with respect to the discussion around the prime number theorem. At $12 it is also a steal! My only criticisms are that it is culturally a little behind the times with references to the student as "he/him" and some strange references to Chinese restaurant menus. Please take these shortcomings in your stride, like you would when chatting to a beloved and wise grandparent who might not have caught up with the times in all the ways that they should have had. This text has a great deal of wisdom and is worth incorporating into your mathematical heart.

A typical day in class The course should be thought of as being similar to one in the humanities (and mathematics is not only the queen of the sciences but very much part of the humanities), in so far as you are very much expected to *come to class with the reading being read*[1].

Class will always open with a short five minute quiz. On each quiz there will be two questions. The first will be a definition or example from the reading due for that day and another will be on a problem assigned for that day. We will then discuss the solutions to the day's quiz and will continue on to the next topic, the topic of the reading that was due that day.

Coming to class without working and thinking hard about the problems and without a first sincere reflection on the assigned reading is a recipe for getting for being completely lost in class (in utilatarian terms, that means spending your semester in that hinterland between course grades D and F) and getting very little out of the course as a whole.

[1]Think of it like a book club where the members actually read about the book!

Quizzes, Journal and Class Participation The quiz questions/problems will be taken directly from the homework. The sheaf of paper that you call "your homework" in other classes will not be handed in, only the quiz. You are expected to have read assigned material *before* class and record your distilled thoughts in a hardbound journal. You can use your journal on some (but not all) quizzes. You can put whatever you wish into your hardback journals but these journals are not for writing in in class and must be separate from the notes that you take in class. You will be able to consult your journal on some but not all quizzes.

Class participation is assigned by points. In short, if you say something that's true and relevant to the discussion you get one point. The most points you can get in any day is two. If more than two-thirds of the class, and I mean *more than two-thirds*, then those who contribute a point to the class discussion get yet another extra point, even those who already have two. Class participation carries 10% and will be added at the end of the semester.

There will be 5% added to your grade at the end of the semester for keeping a good journal on a regular basis. I will look at journals on random days, usually during quiz time.

Tests and final exam There will four in class tests: Jan. 31 (gentle first test), Feb. 28, Apr. 4 (on or about this date, to be confirmed), Apr. 30. The final exam will be a written exam held in Roop 213 on Wed 5/7 from 10.30–12.30.

Assessment As already stated above 15% of the grade is for class participation and journal.

Every quiz and in class tests will contain questions. Questions will be graded on an **A/B/C/D** scale.

- **A:** Excellent and complete solution/argument with the smallest of gaps allowed.
- **B:** A decent, close to complete argument that nonetheless would need one major hint to complete or two minor ones. Questions acheiveing this grade should be re-read carefully with the holes filled in.
- **C:** Proposed argument contains something that is true and relevant. Can also be given to arguments that have serious errors and are not easily fixed.
- **D:** Argument might contain a thread of something that is true and relevant, or be little more than the student's name on the page and/or scribbles.

How to compute your grade? Almost all questions carry equal weight. Occasionally, some test questions might be weighted more. For example, a challenging and/or necessarily long question might be weight double. To get a sense of your overall grade give yourself 5 for every A, 4 for every B, 3 for every C, 2 for every D, 0 for every missed/absent assignment. Compute your average, being sure to count quizzes and assignments missed. Account too for the journal and participation points.

If your cumulative average is 4.5 or above then you are guaranteed an A, equal or above 3.6 but less than 4.5 then you are guaranteed a B, equal or above 2.7 but less than 3.6 is guaranteed a C, and a D is between 2.0 and 2.6. Plus/minus grades will be assigned accordingly.

General JMU policy Go to `www.jmu.edu/syllabus` for university wide policies on Attendance, Academic Honesty and SafeAssign, Adding/Dropping Courses, Disability Accommodations, Inclement Weather and Religious Accommodations.

Introductory Problems

Please see the fourteen questions on Pages 47-48 of the text. It would be very good to use a computer but you do not have to do so. It would be smart to team up with someone who has already completed Math 248 or has a little experience in coding. These questions with your conjectures and proofs as to what is actually happening are due for Tuesday, 1/21 at 10am and can be slid under my office door, Roop 323.

Here are clarifications on what some of the questions are asking:

(1) An example of $d(n)$: $d(12) = 6$ since 12 has 6 integers (including 12 itself) that divide it: $1, 2, 3, 4, 6, 12$

(2) Prime numbers are ones whose only divisors are 1 and itself. 59 is prime. 91 might "look like" a prime but it is not!

(3),(4),(5) Perfect squares are integers of the form n^2 where n is an integer. For example, $1, 4, 9, 16, 25, 36, 49, 64, \ldots$ etc. Examples of a sum of two perfect squares is $2 = 1 + 1 = 2$,$10 = 1 + 9$, $13 = 4 + 9$, etc Similarly, 23 is not a perfect square, not a sum of two or three perfect squares. It is however, a sum of four perfect squares: $23 = 1 + 4 + 9 + 9$. Can you find a number that is not a sum of four or less perfect squares?

(6),(7) The notation $\mid$ and $\nmid$ should be read as "does divide" and "does not divide" respectively. For example $4 \mid 12$ but $4 \nmid 23$.

(8) Just to put more flesh on the bones: 7 can be written as the sum of distinct positive integers in 5 ways, namely: $7, 6 + 1, 5 + 2, 4 + 3, 1 + 2 + 4$.

(9) Just to put more flesh on the bones: 7 can be written as the sum of odd positive integers in 5 ways, namely: $7, 3 + 3 + 1, 5 + 1 + 1, 3 + 1 + 1 + 1 + 1, 1 + 1 + 1 + 1 + 1 + 1 + 1$.

(10) An example of $\sigma(n)$: $\sigma(12) = 30$ since 12 has 6 integers (including 12 itself) that divide it: $1, 2, 3, 4, 6, 12$ and the sum of these integers is $1+2+3+4+6+12 = 30$. Note that number for which $\sigma(n) = 2n$ are called *perfect*: numbers that are equal to the sum of their *proper divisors* (those divisors of the number that are not the number itself). Can you find perfect numbers from your investigations?

(11) Two numbers are *relatively prime* if they have no common divisor. For example, any two prime numbers are relatively prime to each other. Other pairs include 15 and 8; 24 and 91, etc. In contrast, the numbers 12 and 18 are not relatively prime as they both have 3 as a divisor. In fact, they also have 6 as a divisor, which is the largest or *greatest common divisor* of 12 and 18. We denote this by $\gcd(12, 18) = 6$.

(12),(13),(14) Similar to discussion on (8) and (9).