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Class Meetings MWF in Miller G004. §1: 125–215, §2: 230–320.

Office Hours M 9:00-9:45, W 12:20 - 12:50, F 10:10-10:45, or by appointment.

Texts and materials

(1) *Integers, Polynomials, and Rings* by Irving (ISBN: 9780387201726).

(2) *Abstract Algebra: Theory & Applications* by Judson (ISBN: 9781944325022) (Free online version available at http://abstract.pugetsound.edu/aata/aata.html).

(3) There may also be related articles and excerpted chapters posted on the course's canvas website (or handed out in class) that are required reading.

Course Content

This course is an introduction to groups, rings and fields. Topics on rings and fields to be covered include the fundamentals of the integers – a deeper look at modular arithmetic, the division algorithm, the primes, as introduced in math 245 – and the fundamentals of polynomials – their factors and roots over different fields including the fundamental theorem of algebra and criterion for divisibility over $\mathbb{Q}$, polynomial rings and fields and possibly some field extensions, especially quadratic and the link to classical geometry. As for groups, we will cover the basics of subgroups, cyclic groups, some matrix groups with symmetry, permutation and dihedral groups, the group of isometries on the plane, Lagrange's Theorem, Cayley's Theorem, homomorphisms, normal subgroups, the group isomorphism theorems.

The above topics represent the tip of the iceberg of abstract algebra. Due to time constraints we will not get to explore topics like general field extensions and its link to linear algebra, Galois Theory, Gröbner bases, algebraic number theory, coding theory, symmetry groups of regular polyhedra, factor groups, the Sylow Theorems. These topics will be covered in Math 431 but there will be group projects that explore these. Math 431, the depth follow-up to this course, which will be taught in Spring 2018.

Instructional Method

There will be a large emphasis placed on reading, writing, and oral presentation. Most class time will be spent working in groups on structured team assignments. The team assignments will emphasize "play and exploration" as much as "correct proof" so these will not be graded on an A/B/etc. manner. More on this later. There will be a reading quiz implemented through `canvas` that will be due before each class, true/false questions asking mostly about

definitions and examples related to the text. You are expected to work outside of class in teams in a focused intentional manner. Teams may change as the semester progresses.

The Big Picture

The words "abstract" and "algebra" are utilized as accusation and torture respectively, at least in the popular imagination. where they conjure up images aloofness, detachment and years of motion without motive through high school. The good news is that this course is not your high school algebra course; we will not spend a whole semester "solving for x" even if there will be occassions when we will do so.

So what is it about then? Let's call upon your experience in linear algebra for our first intuitive sense. First, you had a lot of definitions and these definitions summarised succinctly many great ideas. Second, you were talking about algebra in a formal sense but what you were really doing was using algebra to understand geometry. There was some computation but what you really spent your time thinking about were matrices as both an object/noun to be studied in its own right, (say, when you asked for solutions to a system of linear equations) and as a action/verb like when you thought of the matrix as a function in its own right that mapped a vector in one vector space to another. Thinking of the matrix as an action is when the subject became really rich, when you started thinking about abstract results like the rank-nullity theorem and eigenvalue/vector problem. This echo of linear algebra will chime true in this course, especially when we talk about groups.

One way to think of groups is that they are the mathematical ideas that codify in a simple elegant way what we mean when we talk about symmetry. We all know symmetry when we see it but defining and modeling what we really mean is an entirely different matter requiring great care and attention to detail. Groups will come in the second half of the course with rings and fields coming in the first half. The good news is that you already have a good intuitive sense of what rings and fields are because you have been thinking about integers and polynomials for a long time. Indeed, we will think very deeply about the simple properties of the integers and polynomials and abstract these ideas to more general settings.

Our course structure will be a little unusual in that we will start with rings and fields and then go onto groups. Most abstract algebra courses start with groups and go more in depth in groups. We are following this model because an enduring value of James Madison University is our tradition of training future teachers and the needs of future teachers are quite specific, that they have a deep mastery of the integers and polynomials and are able to write, converse, and listen on these topics with formidable expertise. For this reason we start with rings and fields first and we explore these topics with a relentless emphasis on

team-based discussion. Groups, also crucial for future teachers (but less so than rings and fields) will come in the second half.

Assessment

- Canvas quizzes will be assigned regularly and are due via canvas every meeting day at 12pm. They are based on the reading assignments and consist of multiple choice questions based on new definitions and examples. These are to be attempted individually and are open book.
- There are four in class tests, scheduled for **W 9/20**, **F 10/13**, **W 11/8**, and **F 12/8**. These are all closed book and the content will be close to team assignments and homework assignments. Each test will only cover new material since the last test, with the caveat that all new material builds on the old.
- Team assignments are conducted in class, usually via a worksheet that is made available before class. These will be collected at the end of each class and are graded on an 9/10 scale. 10 is not for "fully correct" but rather for full engagement and willingness to play with the topics of the day. 9 means you fought but last the round valiantly. 8 is for your heads not showing up, which includes not doing the reading. Questions and conjectures are valued as much if not more than final answers.
- Final Oral Exam for each student will take place during finals week.
- You will be asked to evaluate your own contributitions to the group's work and, intermittently, your group members will be asked to evaluate your contributions to the group. This will affect your score from team assignments. The table below shows what each assessmnent component is worth.

Pre-Class Canvas Quizzes	20%
In Class Tests	60 %
In Class Worksheets	10 %
Final Oral Exam	10%

First Week Attendance Policy

At the instructor's discretion, any student registered for a class in the Department of Mathematics and Statistics who does not attend at least one of the first two scheduled meetings of the class (or does not attend the first scheduled meeting of a class that meets once a week) MAY be administratively dropped from the class. Students will be notified by e-mail if they will be dropped. Students who fail to attend should not assume they will be administratively dropped by their instructor; it is the students responsibility to drop the course on their own

or they will receive a grade at the end of the semester. All students are responsible for verifying the accuracy of their schedules and changes made in their schedules.

General JMU policy

Go to `www.jmu.edu/syllabus` for university wide policies on Attendance, Academic Honesty and SafeAssign, Adding/Dropping Courses, Disability Accommodations, Inclement Weather and Religious Accommodations.