The Broad Questions (or what the final exam is going to look like)

IN ALL SECTIONS

Understand Problems 1 and 2 and revise the "Concepts" problems assigned as HW.

CHAPTER 0 This should be mostly recap. There will be a strong emphasis placed on the domain and range of a function, when a function is one-to-one and to the inverse of a function.

SECTION 1.1 This really sets the table for what's to come. Subsumed by the remainder of the chapter. Note that vertical and horizontal asymptotes are defined here as limits.

SECTION 1.2

- Be able to state the careful $\varepsilon - \delta$ definition of the limit and the cases where either the function and/or the x is tending to infinity. e.g. 19-42.
- Given a fixed epsilon for a claim limit, using algebra and graphs provide a particular largest delta e.g. 43-64.
- Show that a limit is not as stated by finding an epsilon for which no delta exists. e.g. 18.

SECTION 1.4

- Understand the definition of continuity at a point (including left and right continuity), at an interval and what we mean when we say a function is plain continuous. Note that saying that a function f is continuous at $x = c$ is really saying that $\lim_{x\to c} f(x) = f(\lim_{x\to c} x) = f(c)$. In other words, continuous functions are ones where, in colloquial terms, the limit of the function is the function of limit. Distinguish between removable, jump and infinite discontinuities. e.g. 27-23, 39-44.
- Understand the statements of EVT and IVT and be sensitive to their hypothesis. e.g. 49-54,61-66.
- Apply IVT in different types of scenarios. e.g. 76-79. Use the bisction method (a special case of IVT) to locate roots of functions. e.g. 80

SECTION 1.5 Basic rules of limits. Cancellation and squeeze rule for limits. Especially important for coninuity of exponential and trigonometric functions where we define the number e. With the exception of the squeeze rule (e.g. 79-86) many of the insights here occur frequently when evaluating limits in Section 1.6 and when we evaluate derivatives later.

SECTION 1.6 Indeterminate forms of limits. Pay special attention to dividing above and below by the dominating power when $x \to \infty$ (e.g. Example 1 and Problems) and to the trigonometric limits of Theorem 1.35. e.g. 35-80.

SECTION 2.1 This really sets the table for what's to come. Subsumed by the remainder of the chapter.

SECTION 2.2

- Be able to state the careful limit definition of the derivative, both h and z definitions. Compute derivatives of simple functions from the limit defintion. Find the derivative of functions involving roots (using the conjugate) e.g. 39-54.
- Understand what it means for a function to be differentiable (at a point and on an interval). Understand why a function being differentiable implies it is continuous and why the converse does not hold. e.g. Understand the derivatives of a piecewise function. e.g. concepts questions, 69-78.
- Tangent lines and local linearity, Newton's Method for approximating roots. e.g. 59-64, 81-86.
- This section is a first opportunity to acquaint oneself with the $\frac{d}{dx}$ notation for the derivative. This phrasing illuminates the chain rule to come later.

SECTION 2.3 Understand the derivation from definition of the derivative of the power, product and quotient rules. Apply these in various scenarios e.g. 29-64, 65-72.

SECTION 2.4 Chain rule (e.g. 21-46) and Implicit differentiation (e.g 81-84). Wow!

SECTION 2.5

- Derivatives of exponential and log functions. Note especially how the derivative of log is derived not directly from the limit definition of the derivative but rather from the chain rule combined with the fact that exponential and log functions are inverses of each other. e.g. 17-44.
- Exponential functions are one where the rate of change of a quantity is proportional to the quantity itself. e.g. 64-67.
- Logarithmic differentiation (see Examples 4 and 5) e.g. 49-58.

SECTION 2.6 Trigonometric and inverse trigonometric derivatives.

- Understand how the derivative of $\sin(x)$ must be derived from the limit defintion of the derivative. Be able to say how the derivatives of the other trigonometric functions, when combined with chain rule and other such rules, can be derived from knowing sin's derivative. e.g 17-50.
- Understand how to find the deriviatives of the inverse of the trigonometric functions, noting the similarity with exp and log, in so far as, one can use the chain rule on the composition of a trig function and its inverse. Wow! e.g. 17-50.

SECTION 3.1 Understand hypotheses and conclusions that can be drawn from Rolle's Theorem and Mean Value Theorem e.g. 53-60

SECTION 3.2 Subsumed by Section 3.3

SECTION 3.3 First and Second Drivative Tests: Know how thse tests work, how properties of a function f increasing/decreasing are reflected in the derivative of f, same for concavity with f''. What is the distinction between a critical point as opposed to a local max/min. Similarly, what is an inflection point and what does it mean for the graph of f.

- Given a graph of one of f, f', f'' be able to draw a graph of the others of f, f', f''. e.g. Section 3.3: 21-28.
- Given the sign charts of f, f', f'' sketch a possible graph of f e.g. Section 3.3: 59-62.

- Sketch careful labeled graphs of a given function f vis-a-vis Section 3.3: 63-82.

SECTION 3.4 Optimization: The first challange with these problems is to characterise what the question is asking for, what is to be maximized/minimized? what are the constraints given? These can sometimes be quite subtle and require a good deal of clear thinking. One constraint that is often (and erroneously) overlooked is the endpoints on which the function to be (max/min)imized. For example, when one is studying the classical "farmer's rectangular pen" (see first example in Section) of maximizing the pen's area given that the pen has a fixed perimeter, then the smallest that the physical width of the pen can be is zero, and the most the width can be is half of the perimeter.

The second challenge is that even when one uses the first and second derivative tests, these only provide *local* max/mins and one must test which of these are the global maxima. Sometimes it requires that you remember the formulae for surface areas and volumes of cones, cylinders, &c.

Revise all the problems that we covered when discussing this section in class. There might be a problem that asks you to formulate and fully solve an optimization problem, there might be one where you only have to formulate, or one where the formulation is given and where all you must do is find a global max or min.

SECTION 3.5 Related Rates: The challenge here is much the same as that of the Optimization problems, that is, translating the problem in English into a concise mathematical statement. It is always helpful to organise your understanding of the problem, as we did when covering this section, into four categories: a decent *picture*, a *known* rate of change, a *desired* rate of change and the *relationship* between the variables in the known and desired rates of change. Revise all the problems that we covered when discussing this section in class.

SECTION 3.6 L'Hopital's Rule: Understand statement of the rule, and how it was proven using local linearity (Problems 21-48). Using the logarithm to evaluate limits like $\lim_{x\to 0^+} x^x$. (49-64).

SECTION 4.1 Addition and Accumulation (The Σ notation) is mostly subsumed by Riemann sums. Be sure to note Theorem 4.4. In an exam, it would be reasonable to ask a soft question like those in 21-28.

SECTION 4.2 Riemann Sums:

- Appoximate the area between the x-axis and a curve $y = f(x)$, on $[a, b]$ using left, right, midpoint and trapezoid rules. e.g. 27-33.
- Write a Riemann sum approximation in sigma notation e.g. 39-44.
- Given an expression for a Riemann sum approximation in sigma notation, recognize the function and $[a, b]$ e.g. 17-20.

SECTION 4.3 Definite integrals:

- Understand that the definition of the definite integral is the limit of a Riemann sum.
- Find the exact value of a definite integral by setting up a Riemann sum approximation and taking its limit e.g. 41-46.
- Utilize properties of the definite integral e.g. 29-40.

SECTION 4.4 Indefinite Integrals: It's the only price one has to pay to use the First FTC e.g. 21-58.

Section 4.5 FTC I:

- Understand that the statement of FTC I. A familiarity with the key ideas of its proof, e.g. MVT and telescoping sums.
- Compute signed areas using FTC I e.g. 19-64.

Section 4.6 Absolute (farmer's) and signed (Riemann's) area ("average values" covered if time permits):

- Compute the absolute area e.g. 20-37
- Compute (absolute) area between curves e.g. 41-52.

Section 4.7 FTC II:

- Understand the statement of FTC II
- Differentiating area accumulation functions and compositions involving area accumulation functions e.g. 27-48.