
Problem of the Week

Solution Five



Here is what is shown by the first three scales:

1. One glass plus one bottle balances one pitcher.
2. One bottle balances one glass plus one plate.
3. Two pitchers balances three plates.

The question at the bottom reads: How many glasses will balance with the bottle?

SOLUTION: From the third diagram, we see that two pitchers balance three plates, so we know that one plate is equal to $\frac{2}{3}$ of a pitcher.

If we add a glass to each side of the second illustration, then the scale will still balance. But now the left arm of the second scale is the same as the left arm of the first scale. Combining the first two scales now shows us that one pitcher is the same as one glass and one bottle, which is the same as two glasses plus one plate.

Since a plate is $\frac{2}{3}$ of a pitcher, the two glasses must make up the other $\frac{1}{3}$. We conclude that each glass is $\frac{1}{6}$ of a pitcher.

From the first illustration we see that a glass (which is $\frac{1}{6}$ of a pitcher) and a bottle balance with a pitcher. That tells us that a bottle must be $\frac{5}{6}$ of a pitcher. Therefore, to balance the bottle we need five glasses.