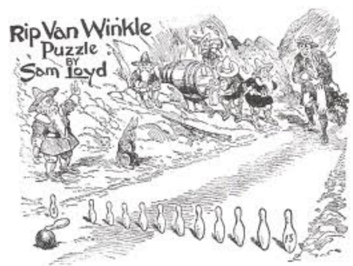
Problem of the Week
Solution Eight



The old Dutch game of *Kugelspiel*, from which modern bowling is derived, used to be played with thirteen pins placed in a row. Only one or two (adjacent) pins could be knocked down by any single shot. The bowlers stood so close to the pins that it did not call for much skill to hit any single pin desired, or any two adjacaent ones. Players bowled alternately, one ball at a time, and the point of the game was to see who could knock down the last pin.

The little Man-of-the-Mountain, with whom Rip Van Winkle is playing this game, has just rolled a ball and knocked out pin No. 2. Rip has a choice of twenty-two different plays: any one of the twelve single pins, or any one of the ten open spots that will bring down two adjacent pins. What is Rip's best shot to win the game? It is assumed that both players can hit any pin or pair of pins they wish, and that there is the best possible play on both sides.

SOLUTION: This is a version of a puzzle that is discussed in H. E. Dudeney's book *The Canterbury Puzzles*. His solution is quite lucid, so I will repeat it verbatim:

> To win at this game you must, sooner or later, leave your opponent an even number of similar groups. Then whatever he does in one group you repeat in a similar group. Suppose, for example, that you leave him these groups:
>
> $$(0 \quad 0 \quad 000 \quad 000)$$
>
> Now, if he knocks down a single, you knock down a single; if he knocks down two in one triplet, you knock down two in the other triplet; if he knocks down the central kayle in a triplet, you knock down the central one in the other triplet. In this way you must eventually win. As the game is started with the arrangement
>
> $$(0 \quad 00000000000),$$

the first player can always win, but only by knocking down the sixth or tenth kayle (counting the one already fallen as the second), and this leaves in either case

$$(0 \quad 000 \quad 0000000)$$

as the order of the groups is of no importance. Whatever the second player now does, this can always be resolved into an even number of equal groups. Let us suppose that he knocks down the single one; then we play to leave him

$$(00 \quad 0000000).$$

Now, whatever he does we can afterwards leave him

$$(000 \quad 000) \quad \text{or} \quad (0 \quad 00 \quad 000).$$

We know why the former wins, and the latter wins also; because, however he may play, we can always leave him either

$$(0 \quad 0) \quad \text{or} \quad (0 \quad 0 \quad 0 \quad 0) \quad \text{or} \quad (00 \quad 00)$$

as the case may be. The complete analysis I can now leave for the amusement of the reader.