Problem of the Week
Number Four
September 22, 2014

There's an old story about the time when the great mathematician G. H. Hardy visited his friend and fellow mathematician Srinivasa Ramanujan in the hospital. Apparently Hardy noted that the number of the taxi he rode in was 1729, and he remarked that this was not such an interesting number. Ramanujan immediately admonished him. "Not so," he said. "It is a very interesting number. It is the smallest positive integer that can be expressed as the sum of two cubes in two different ways. Indeed! You might amuse yourself by verifying that

$$\begin{aligned} 1729 &= 1^3 + 12^3 = 1 + 1728 \\ &= 9^3 + 10^3 = 729 + 1000. \end{aligned}$$

To be strictly accurate, however, Ramanujan really ought to have stipulated that he meant the sum of two *positive* cubes. If we allow negative cubes then we have this:

$$\begin{aligned} 91 &= 3^3 + 4^3 = 27 + 64 \\ &= 6^3 + (-5)^3 = 216 - 125. \end{aligned}$$

I wonder what would have happened had the number on the taxi been 40585. Would Ramanujan have noticed that

$$40585 = 4! + 0! + 5! + 8! + 5!$$

The exclamation point indicates a *factorial*. If n is a positive integer, then $n!$ means to multiply all of the positive integers from 1 up to n. So, for example,

$$\begin{aligned} 4! &= 1 \times 2 \times 3 \times 4 = 24 \\ 5! &= 1 \times 2 \times 3 \times 4 \times 5 = 120. \end{aligned}$$

For convenience, we make the definition $0! = 1$
It turns out that there are exactly four positive integers that are equal to the sum of the factorials of their digits. One is 40585, as we have just seen. There are also the trivial cases $1! = 1$ and $2! = 2$. And the fourth? Well, that's this week's problem!

Find a three-digit number that is equal to the sum of the factorials of its digits.

A small amount of trial and error is inevitable on this one, but with some cleverness you can hone in on the correct answer pretty quickly. When you think you have it, follow the directions on the other side of this page $\Longrightarrow$

Solutions are due to Jason Rosenhouse by 5:00 on Friday, September 26. ***Solutions should be written on an official POTW handout, in the space below.*** *Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the* ***upper right corner*** *of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Please make sure that the answer to the problem is displayed clearly and prominently, in a box when appropriate. Problems are available at the bulletin board outside Roop 119, and also at the website:*

http://educ.jmu.edu/∼rosenhjd/POTW/Fall14/homepage.html