
Problem of the Week

Solution Four

Find a three-digit number that is equal to the sum of the factorials of its digits.

SOLUTION: The correct answer is 145. We have

$$145 = 1! + 4! + 5! = 1 + 24 + 120.$$

The first observation is that since $7! = 5040$, there is no way our number contains a digit that is 7 or larger. Since $6! = 720$, we further see that our mystery number cannot contain a digit that is 6 or larger. It follows that we may only use the digits from 0 through 5.

The next observation is that our number must contain exactly one 5. To see this, note first that 555 simply does not work. Then note that if our number contains two 5s, they would have to be the last two digits in the number. This follows from the fact that we are not able to obtain a sum greater than 500 given our restrictions. It is a matter of quick inspection to see that 155, 255, 355 and 455 do not work. Finally, note that since $4!$ is only 24, there is no way to obtain a sum greater than 100 if we use no 5s at all. The conclusion is that our number contains exactly one 5.

Since the other two digits are no greater than 4, the sum of the factorials cannot possibly exceed 199. It follows that the first digit in our number is 1. Since we know that one of the other digits is 5, we only need to check a small number of possibilities before stumbling onto 145.