Problem of the Week
Solution Five

In the addition problem below, each letter stands for a different digit. However, each letter stands for the same digit in every place where it appears. Find the only possible value for each digit to make a correct addition statement:

$$\begin{array}{rccccc} & & S & E & N & D \\ + & & M & O & R & E \\ \hline & M & O & N & E & Y \end{array}$$

The first observation is simply that we must have $M = 1$. (Note that we don't allow 0 to be the first digit of a number.) Two four-digit numbers cannot sum to something greater than $20{,}000$. So, right off the bat, we have this:

$$\begin{array}{rccccc} & & S & E & N & D \\ + & & 1 & O & R & E \\ \hline & 1 & O & N & E & Y \end{array}$$

The next observation is that either $S = 9$ or $S = 8$ with a carry of one from the previous column to make a sum greater than ten. In either case, we see that $O = 0$. Our problem now looks like this:

$$\begin{array}{rccccc} & & S & E & N & D \\ + & & 1 & 0 & R & E \\ \hline & 1 & 0 & N & E & Y \end{array}$$

Now it is clear that there cannot be a carry from the third to the fourth column, meaning that $S = 9$:

$$\begin{array}{rccccc} & & 9 & E & N & D \\ + & & 1 & 0 & R & E \\ \hline & 1 & 0 & N & E & Y \end{array}$$

Since we cannot have that $E = N$, we must have a carry from the second column to the third. It follows that $N = E + 1$.

This is where things get tricky. Let's suppose there is no carry from the first column to the second. In this case, we would have $N + R = 10 + E$, since we need a carry into the third column.

Substituting for N in this equation gives $(E+1)+R = 10+E$. This implies that $R = 9$, which is impossible since we have already determined that $S = 9$.

Thus, we must have a carry from the first column into the second. That means that $N + R + 1 = 10 + E$. Once more making our substitution gives us $(E+1)+R+1 = 10+E$, which immediately gets us that $R = 8$. Our problem now looks like this:

$$\begin{array}{ccccc} & 9 & E & N & D \\ + & 1 & 0 & 8 & E \\ \hline 1 & 0 & N & E & Y \end{array}$$

Now, since we must have a carry from the first column into the second, we see that $D+E = 10+Y$. Since 0 and 1 are already taken, it must be that $D+E$ is at least 12. Given the digits that remain available, the only possibilities are that D and E are 5 and 7, or 6 and 7. In either case, one of them is 7. We certainly cannot have that $E = 7$, since then $N = 8$, which is impossible since 8 is already taken. It follows that $D = 7$.

Now we see we cannot have $E = 6$, since then $N = 7$, which is impossible since we know $D = 7$. The only way out is to suppose that $E = 5$ and $D = 7$. The rest of the letters now fall immediately, and we have the solution:

$$\begin{array}{ccccc} & 9 & 5 & 6 & 7 \\ + & 1 & 0 & 8 & 5 \\ \hline 1 & 0 & 6 & 5 & 2 \end{array}$$