Problem of the Week
Number Ten
November 30, 2015

Our final "false proof" for the term is of a more philosophical cast. It feels more like a word game than a proper mathematical proof. So how about we consider a few warm-up puzzles that involve unraveling some convoluted language?

(1) You see a man looking at a portrait. Pointing to the person in the portrait, the man says, "Brothers and sisters I have none, but this man's father is my father's son." Who is the person in the portrait?

(2) That one's such a classic you might have heard it before. So try this one: How many people do you have if you have two pairs of twins twice?

(3) Want more? Well, suppose you ask me on Friday what day classes start, and I truthfully reply that they start two days after the day before the day after tomorrow. What day do classes start?

(4) Still not enough? Then try this: The Supreme Court today reversed its earlier ruling that let stand an appellate court's decision to overturn a lower court's finding that a restaurant owner had no right to fire a waiter for refusing to deny service to a male patron who was not wearing a tie and jacket. If a male patron now enters that restaurant without a tie and jacket, and if we assume the wait staff will serve anyone so long as they are confident they will not be fired for doing so, then will the patron be served?

Food for thought, all. But none of them is actually the Problem Of the Week. That comes next, between the lines, but a little set-up is called for. When I speak of "unambiguous descriptions in fourteen words or less," I have in mind any string of words that you might find in an English dictionary. For example, "ten cubed" is an unambiguous description of the number 1000, and "the smallest prime number larger than one hundred" is an unambiguous description of the number 101. Since there are only finitely many phrases that can be built from fourteen words or less, but there are infinitely many numbers, common sense dictates that there must be some numbers (infinitely many, in fact) that cannot be unambiguously described by such phrases. But the "proof" below disputes that conclusion:

I am going to prove that every natural number can be unambiguously described in fourteen words or less. Assume for a contradiction that there is a natural number that can not be so described. Then there must be a smallest such number. Call that number n. Then n can be unambiguously described by the phrase, "the smallest natural number that cannot be unambiguously described in fourteen words or less." That phrase has fourteen words. Thus, we have reached a contradiction. It follows that the number n cannot exist, and our claim is proved.

Plainly, something has gone wrong. But where, *precisely*, does the error lie? When you have a good answer to that, follow the instructions on the back of this page $\Longrightarrow$

SEE YOU NEXT TERM!

*Submissions are due to Jason Rosenhouse by 5:00 on **Friday, December 4**. Solutions should be written on the back of an official POTW handout. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the **upper right corner** of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Answers will be judged on the clarity with which they explain the flaw in the argument. Solutions will be posted at this website, by the Monday after the problem is due:*

http://educ.jmu.edu/∼rosenhjd/POTW/Fall15.html