# Problem of the Week
# Number One
# September 7, 2015

Here's a question for you: Given the equation

$$\frac{1}{4}\text{ Dollar} = 25\text{ Cents},$$

doesn't it follow, by taking square roots of both sides, that

$$\frac{1}{2}\text{ Dollar} = 5\text{ Cents?}$$

Or how about this one: Did you know that it is possible to house thirteen people in twelve rooms, with no two people having to share a room? Start by temporarily putting the first and last person in room one. Then put the third person in room two, the fourth in room three, the fifth in room four, and so on until the twelfth person is placed in room eleven. But now room twelve is free is for the thirteenth person, who can now be removed from room one. Mission accomplished!

Those are examples of our theme for this semester:

**FALSE PROOFS**

We're talking about those devious mathematical arguments that seem plausible at first sight, but lead to plainly absurd conclusions. Each week I shall present one of these false proofs. Your task, dear problem solver, is to explain, *with crystal clarity*, exactly where the proof went wrong. Students, you can think of this semester as the revenge of your math professors. How much time do we spend each term chasing through your dubious arguments, trying to pinpoint the moment where you went off the rails? Now you get a taste of your own medicine. Ha!

Our first two examples were, perhaps, so crass that spotting the flaw poses little challenge. But sometimes the error can be maddeningly subtle and well-concealed. Sometimes it can feel like a magic trick, as though with a spot of misdirection I made you think you had seen something impossible. So sit back and enjoy the show!

Let's kick off the festivities with one of the slam dunk classics of the genre:

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I'm going to prove that an elephant weighs the same as a fly. Let $e$ denote the weight of an elephant and let $f$ denote the weight of a fly. Let us denote by $2s$ the sum of these weights. (Note that $e$, $f$, and $s$ are not necessarily whole numbers). We then have that $e + f = 2s$. This implies that $e - 2s = -f$ and $e = -f + 2s$. Multiplying these equations together gives us

$$e^2 - 2es = f^2 - 2fs.$$

Adding $s^2$ to both sides gives

$$e^2 - 2es + s^2 = f^2 - 2fs + s^2,$$

which implies that

$$(e - s)^2 = (f - s)^2.$$

Taking square roots now gives us that $e - s = f - s$. This implies that $e = f$ as claimed!

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Mull that one over for a while, and when you think you've spotted the flaw follow the instructions on the other side of the page $\Longrightarrow$

*Submissions are due to Jason Rosenhouse by 5:00 on* ***Friday, September 11****. Solutions should be written on the back of an official POTW handout. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the* ***upper right corner*** *of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Answers will be judged on the clarity with which they explain the flaw in the argument. Solutions will be posted at this website, by the Monday after the problem is due:*

**http://educ.jmu.edu/∼rosenhjd/POTW/Fall15.html**