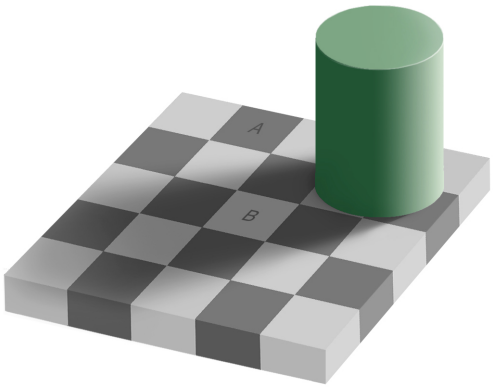
Problem of the Week
Number Three
September 21, 2015

Optical illusions can be a lot of fun. Here's one from real life:

That's the famous Gateway Arch in St. Louis. It's hard to believe just from looking at it, but its height is exactly the same as its width.

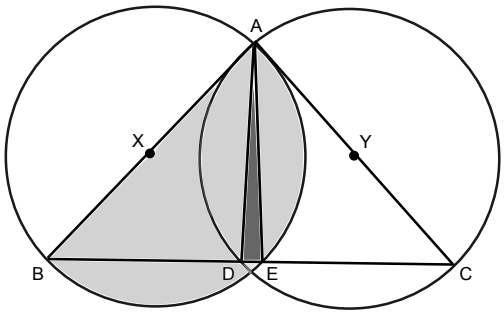
A few minutes with Google will lead you to a great many fiendish illusions to ponder. Surely, though, one of the most astonishing is this one:



Since I introduced this as an illusion, you can probably guess what's coming. The squares A and B are exactly the same shade of grey. I'm not kidding. They're the same. Really. Just use whatever strips of paper you have handy to cover up everything in the diagram except for those two squares. You can also type "Checkershadow Illusion," into Google. The first link will lead to a website that explains everything.

Is any of that relevant to this week's problem? You decide. Here's this week's false proof:

I'm going to show that it is possible to construct a triangle with two right angles:



We have two overlapping circles. One has its center at X, while the other has its center at Y. We draw diameters AB and AC. Then we connect points B and C. This line segment intersects one circle at point D and the other at point E. The result is the tall, thin triangle ADE, shaded dark grey in the figure.

Now notice that angle AEB (which is the same as angle AED) is inscribed in semicircle $AEBX$, shaded light grey in the figure. It is a standard theorem in geometry that an angle inscribed in a semicircle is a right angle. So angle AEB is a right angle, and therefore so is angle AED. Likewise, angle ADC is inscribed in semicircle $ADCY$. Therefore, ADE is a right angle as well. It follows that AED is a triangle with two right angles, which shows that such a thing is possible. $\Longrightarrow$

Isn't that cool? Too bad it's nonsense. When you think you've located where it all went wrong, please explain it as clearly as possible and then follow these instructions.

Submissions are due to Jason Rosenhouse by 5:00 on ***Friday, September 25****. Solutions should be written on the back of an official POTW handout. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the* ***upper right corner*** *of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Answers will be judged on the clarity with which they explain the flaw in the argument. Solutions will be posted at this website, by the Monday after the problem is due:*

http://educ.jmu.edu/∼rosenhjd/POTW/Fall15.html