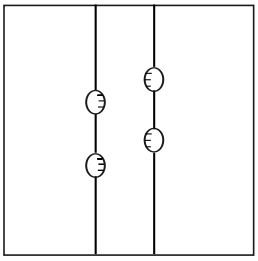
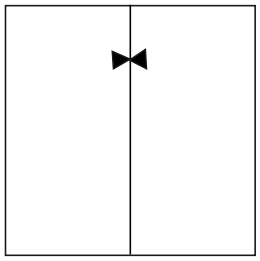
Problem of the Week
Number Five
October 5, 2015

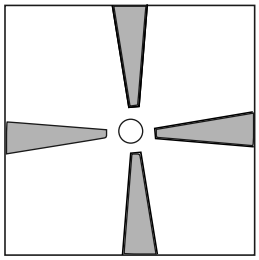
We have another geometric teaser for you this week. Since geometry problems require looking carefully at diagrams, how about we start with a few Droodles.

Droodles are like Rorschach tests in that you are free to read into them whatever you want. They are generally very simple diagrams with just a few abstract shapes and objects. Your task is to come up with an appropriate caption, the sillier and more humorous the better. For example, shown this Droodle:



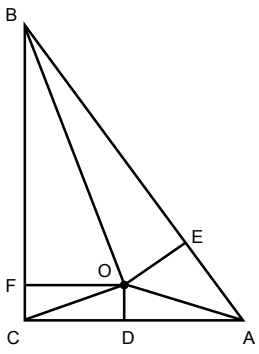
you might decide it is a rear view of a koala bear climbing a tree. Get the idea? I hope so, because here are two more:





Make what you will of those, and then give some thought to this week's false proof. This is a toughie, so don't give up to easily!

I will now prove that the hypotenuse of a right triangle is equal to one of its legs. We start with triangle ABC, with right angle at C.



We draw the angle bisector from B and the perpendicular bisector of AC, and we denote by O the point where they intersect. (Thus, we have that $\angle CBO = \angle ABO$, $CD = AD$, and $\angle ODA = \angle ODC = 90$.) We also draw segment OF perpendicular to BC and segment OE perpendicular to AB.

We now observe that $\triangle BOE \cong \triangle BOF$. Both are right triangles, they have a common side, and their angles at B are equal since OB is an angle bisector. It follows that $BE = BF$.

We also know that $\triangle OFC \cong \triangle OEA$. To see this, notice first that both are right triangles. Then note that since OD is a perpendicular bisector, we have that $OC = OA$. Finally, our previous triangle congruence establishes that $OF = OE$. Since these triangles are congruent, it follows that $EA = FC$.

We conclude that

$$BE + EA = BF + FC.$$

This is equivalent to the claim that $BA = BC$, which is to say that the hypotenuse of the triangle is equal to one of its legs.

There is a lot to wade through in that one, but it is clear that *something* has gone wrong. When you think you have found what it is, follow the instructions on the other side of this page $\Longrightarrow$

*Submissions are due to Jason Rosenhouse by 5:00 on **Friday, October 9**. Solutions should be written on the back of an official POTW handout. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the **upper right corner** of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Answers will be judged on the clarity with which they explain the flaw in the argument. Solutions will be posted at this website, by the Monday after the problem is due:*

http://educ.jmu.edu/∼rosenhjd/POTW/Fall15.html