Problem of the Week
Number Six
October 19, 2015

Since POTW had last week off, I think we need to recharge our batteries. So here's a little arithmetical exercise for you. Add up the following list of numbers, but do it in the following manner: Start from the top and call out a running sum of the numbers as you go, adding in your head. You will start by saying, "One thousand, one thousand ten, two thousand thirty," and so on Sounds easy, right? So give it a try:

$$\begin{array}{r} 1000 \\ 10 \\ 1020 \\ 10 \\ 1030 \\ 10 \\ 1010 \\ 10 \end{array}$$

Further discussion will be deferred to the solutions. You might want to check your answer with a calculator though. The answer might come as a surprise.

Enough of that. It's time to get down to business. Compared to POTW 5, this one should be a walk in the park. But be careful! It can still trip up the unwary.

I am going to prove that $4 = 0$. Consider the following system of equations:

$$\frac{x}{y} + \frac{y}{x} = 2$$
$$x - y = 4.$$

We can multiply the first equation by xy to obtain $x^2 + y^2 = 2xy$. This implies that $x^2 - 2xy + y^2 = 0$ and $(x - y)^2 = 0$. We conclude that $x - y = 0$, and finally that $x = y$.

If we now substitute into the second equation, we obtain $x - x = 4$. But $x - x = 0$. This is possible only if $4 = 0$, as claimed.

Mull that one over for a while, and when you think you've spotted the flaw follow the instructions on the other side of the page $\Longrightarrow$

Submissions are due to Jason Rosenhouse by 5:00 on ***Friday, October 23****. Solutions should be written on the back of an official POTW handout. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the* ***upper right corner*** *of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Answers will be judged on the clarity with which they explain the flaw in the argument. Solutions will be posted at this website, by the Monday after the problem is due:*

http://educ.jmu.edu/∼rosenhjd/POTW/Fall15.html