Problem of the Week Number Nine November 9, 2015

Did you hear the one about the guy who was lost in the woods with no food and no supplies? He screamed until he was hoarse, then he got on the horse and rode out.

That's right! It's a horse theme for our penultimate POTW. We have a classic false proof for you this week, one you may have encountered in a discrete mathematics class. We'll build up to it with a bit of horseplay. We could discuss philosophy instead, but that would be putting Descartes before the horse.

Rim shot!

There were these two horses in a field. They were facing in opposite directions, one looking due north, the other due south. But they were still able to see each other, without employing any mirrors or reflective surfaces. How is this possible?

How about this: Three horses were in a race. If the odds were $2 : 1$ against the first horse winning and $3 : 2$ against the second horse winning, what were the odds against the third horse?

Here's another one: It seems that according to their father's will, three sons were supposed to divide up seven horses. The will called for the oldest son to get half of the horses, for the middle son to get a quarter of the horses, and for the youngest to get one-eighth of the horses. The sons were chagrined, since short of cutting up the horses into little pieces it seemed impossible to fulfill the terms of the will. But then a neighbor came over and added one of his own horses, making a total of eight. The oldest took four of the horses (half), the middle son took two horses (a quarter), and the youngest took one horse (one-eighth). This accounted for the original seven horses. The neighbor then took his own horse back and went on his way. Problem solved!

Maybe it's time to get down to business. See if you can pinpoint the error in *this* argument:

I am going to prove that in any finite set of horses, all of the horses have the same color. To do this, I will employ induction. It is clear that if the set only has one horse, then all of the horses in the set have the same color. That's the base case. Now assume we have proved our result for some arbitrary whole number n. Then consider a set with $n + 1$ horses. Label them $h_1, h_2, \ldots, h_n, h_{n+1}$.

Now, consider horses h_1 through h_n. That's a set with n horses, so we know by the inductive hypothesis that all of these horses have the same color. Likewise, horses h_2 through h_{n+1} must also have the same color. But if h_1 and h_2 have the same color, and h_2 and h_{n+1} have the same color, then it follows that h_1 and h_{n+1} have the same color. So all $n + 1$ horses have the same color.

It now follows by induction that all horses have the same color, as claimed.

Mull that over, and when you think you have the answer let me know. You could go do something else if you prefer, but that would be a horse of a different color. $\Longrightarrow$

Submissions are due to Jason Rosenhouse by 5:00 on ***Friday, November 13****. Solutions should be written on the back of an official POTW handout. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the* ***upper right corner*** *of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Answers will be judged on the clarity with which they explain the flaw in the argument. Solutions will be posted at this website, by the Monday after the problem is due:*

http://educ.jmu.edu/~rosenhjd/POTW/Fall15.html