
Problem of the Week

Solution Ten

(1) You see a man looking at a portrait. Pointing to the person in the portrait, the man says, “Brothers and sisters I have none, but this man’s father is my father’s son.” Who is the person in the portrait?

SOLUTION: Since the man has no siblings, “my father’s son” can only refer to himself. So his statement is really, “This man’s father is me.” So the man is looking at a picture of his son.

(2) How many people do you have if you have two pairs of twins twice?

SOLUTION: A pair of twins is two people. Two pairs of twins is four people. If you have that twice then you have a total of eight people. Some people respond too quickly and think there are four doublings implied by the words “two,” “pairs,” “twins,” and “twice.” This leads them to give the erroneous answer of sixteen.

(3) Suppose you ask me on Friday what day classes start, and I truthfully reply that they start two days after the day before the day after tomorrow. What day do classes start?

SOLUTION: The trick is to work it backward. If today is Friday, then “the day after tomorrow” is Sunday. The day before that is Saturday. Two days after that is Monday. So classes

start on Monday, which is not really so surprising when you think about it.

(4) The Supreme Court today reversed its earlier ruling that let stand an appellate court’s decision to overturn a lower court’s finding that a restaurant owner had no right to fire a waiter for refusing to deny service to a male patron who was not wearing a tie and jacket. If a male patron now enters that restaurant without a tie and jacket, and if we assume the wait staff will serve anyone so long as they are confident they will not be fired for doing so, then will the patron be served?

SOLUTION: Again, the trick is to work it backward. (1) The waiter “refused to deny service” to the poorly dressed patron. That means he served the patron. (2) He was fired for serving the patron. (3) The lower court found the restaurant owner had no right to fire the waiter. (4) The appellate court overturned that decision. At this point the owner had the right to fire the waiter. (5) The Supreme Court initially let this decision stand. (6) The Supreme Court then reversed that decision, meaning that the owner had no right to fire the waiter.

We conclude that the patron will be served, despite not wearing a tie and jacket.

THE MAIN PROBLEM: I am going to prove that every natural number can be unambiguously described in fourteen words or less. Assume for a contradiction that there is a natural number that can not be so described. Then there must be a smallest such number. Call that number n . Then n can be unambiguously described by the phrase, “the smallest natural number that cannot be unambiguously de-

scribed in fourteen words or less.” That phrase has fourteen words. Thus, we have reached a contradiction. It follows that the number n cannot exist, and our claim is proved.

SOLUTION: For convenience, let us denote by S the phrase, “the smallest natural number that cannot be unambiguously described in fourteen words or less.” The problem is that S refers to itself in a logically inconsistent manner. It is a claim about descriptions of numbers, when it is itself such a claim. If you try to apply S to any specific number, then you are effectively claiming that S does not apply to that number. That means that S cannot be regarded as an unambiguous description of any particular number.

We should note, however, that there is a unique number, call it n , such that n is the smallest natural number that cannot be unambiguously described in fourteen word or less. The problem is that S is not an unambiguous description of n .
