
Problem of the Week

Solution One

PROBLEM: Here's a question for you: Given the equation

$$\frac{1}{4} \text{ Dollar} = 25 \text{ Cents},$$

doesn't it follow, by taking square roots of both sides, that

$$\frac{1}{2} \text{ Dollar} = 5 \text{ Cents?}$$

SOLUTION: The problem here is that we only took the square roots of the numbers, and not the units. Indeed, it is very unclear how to take the square root of "Dollars" or "Cents." For a comparison, from the correct equation

$$1 \text{ ft}^2 = 144 \text{ in}^2,$$

it really does follow that

$$1 \text{ ft} = 12 \text{ in}.$$

PROBLEM: Did you know that it is possible to house thirteen people in twelve rooms, with no two people having to share a room? Start by temporarily putting the first and last person in room one. Then put the third person in room two, the fourth in room three, the fifth in room four, and so on until the twelfth person is placed in room eleven. But now room twelve is

free is for the thirteenth person, who can now be removed from room one. Mission accomplished!

SOLUTION: If you reread the problem carefully, you will find that the second person was never actually placed in a room.

MAIN PROBLEM: I'm going to prove that an elephant weighs the same as a fly. Let e denote the weight of an elephant and let f denote the weight of a fly. Let us denote by $2s$ the sum of these weights. (Note that e , f , and s are not necessarily whole numbers). We then have that $e + f = 2s$. This implies that $e - 2s = -f$ and $e = -f + 2s$. Multiplying these equations together gives us

$$e^2 - 2es = f^2 - 2fs.$$

Adding s^2 to both sides gives

$$e^2 - 2es + s^2 = f^2 - 2fs + s^2,$$

which implies that

$$(e - s)^2 = (f - s)^2.$$

Taking square roots now gives us that $e - s = f - s$. This implies that $e = f$ as claimed!

SOLUTION: The problem came in the final step. The equation

$$(e - s)^2 = (f - s)^2$$

does not imply that $e - s = f - s$. Since square roots can be negative, it is possible to have $e - s = -(f - s)$. Indeed, that is precisely the case here. The quantity s represents the average of e and f . The quantities e and f are then equidistant from s , with e being greater than s and f being smaller than s .
