Problem of the Week Solution Nine

PROBLEM: There were these two horses in a field. They were facing in opposite directions, one looking due north, the other due south. But they were still able to see each other, without employing any mirrors or reflective surfaces. How is this possible?

SOLUTION: The horses were facing each other.

PROBLEM: Three horses were in a race. If the odds were $2 : 1$ against the first horse winning and $3 : 2$ against the second horse winning, what were the odds against the third horse?

SOLUTION: It is simplest to convert everything to probabilities. Odds of $2 : 1$ against translate into a $1/3$ probability of winning. Odds of $3 : 2$ against represent a $2/5$ probability of winning. Since the probabilities must sum to one, the probability of the third horse winning is $4/15$. That translates into odds of $11 : 4$ against him.

MAIN PROBLEM: I am going to prove that in any finite set of horses, all of the horses have the same color. To do this, I will employ induction. It is clear that if the set only has one horse, then all of the horses in the set have the same color. That's the base case. Now assume we have proved our result for some arbitrary whole number n. Then consider a set with $n + 1$ horses. Label them $h_1, h_2, \ldots, h_n, h_{n+1}$.

Now, consider horses h_1 through h_n. That's a set with n horses, so we know by the inductive hypothesis that all of these horses have the same color. Likewise, horses h_2 through h_{n+1} must also have the same color. But if h_1 and h_2 have the same color, and h_2 and h_{n+1} have the same color, then it follows that h_1 and h_{n+1} have the same color. So all $n + 1$ horses have the same color.

It now follows by induction that all horses have the same color, as claimed.

SOLUTION: The argument we gave for going from n to $n+1$ is only correct if $n > 2$. If there are exactly two horses in the set, then our trick of overlapping two subsets of size $n - 1$ does not work.

So, the theorem is true when $n = 1$. And if it is true for any $n > 2$ then it is true for all n. But since there is no way of making the jump from $n = 1$ to $n = 2$, the whole proof collapses.