Problem of the Week Number Seven October 24, 2016

Last week we included a Shakespearean sonnet that discussed time. Here's another poem I've always liked:

Once there was an elephant Who tried to use the telephant— No! No! I mean an elephone Who tried to use the telephone— (Dear me! I'm not certain quite That even now I've got it right). Howe'er it was, he got his trunk Entangled in the telephunk; The more he tried to get it free, The louder buzzed the telephee— (I fear I'd better drop this song Of elephop and telephong!)

Maybe that's not your cup of tea, but can we at least agree that poetry should rhyme? I mean, seriously, folks, if you're poetry doesn't rhyme then just write prose and be done with it.

That poem has nothing to do with clocks, of course. So try this teaser:

Jane has two clocks. She sets them accurately to the same time. After an hour has passed, she notices that the first clock gained one minute, while the second clock lost two minutes every hour. We shall assume that the clocks continue to run at a constant rate. The next morning she finds that the first clock reads 7:00 am, while the second clock reads 6:00 am. At what time did Jane set the two clocks?

Food for thought, certainly, but it is not this week's problem. Instead, we have a mixture of clocks and geometry, with a soupçon of trigonometry, for you to consider:

At 6:00, the distance between the tips of the hands of a clock is 41. At 9:00 the distance between the tips of the hands of the same clock is 29. How far apart are the tips of the hands at 10:00?

In working this problem, you might find it helpful to use the law of cosines, If a, b, c represent the lengths of the sides of a triangle, and if θ represents the angle between sides a and b, then we have that

$$c^2 = a^2 + b^2 - 2ab\cos\theta.$$

Of course, in the special case where θ is a right angle, this formula is just equivalent to the Pythagorean theorem.

Submissions are due to Jason Rosenhouse by 5:00 on Friday, October 28. Solutions should be written on the back of an official POTW handout. Provide a brief explanation of your answer. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the upper right corner of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Solutions will be posted at this website, by the Monday after the problem is due:

http://educ.jmu.edu/~rosenhjd/POTW/Spring15.html