Problem of the Week
Solution Five

A man looks at a twelve-hour clock and sees that the minute and hour hands perfectly coincide. He did not realize, however, that on this clock the two hands rotated in opposite directions. If we assume that the correct time was between 4:00 and 5:00, and that the two hands started together at noon, then what is the correct time? You should round your answer to the nearest minute.

SOLUTION: Rounded to the nearest minute, the correct time is 4:37.

Let us first assume that the hour hand continues to move in the normal, clockwise, direction, while the minute hand now moves counterclockwise. When the correct time is 4:00, this clock will in fact read 4:00, just like a normal clock. After x minutes have elapsed, on this clock the minute hand will be pointing at minute mark $60 - x$. The hour-hand will be pointing at minute mark $20 + \frac{x}{12}$. We now do some algebra:

$$\begin{aligned} 60 - x &= 20 + \frac{x}{12} \\ 40 &= \frac{13x}{12} \\ 480 &= 13x \end{aligned}$$

This leads to $x = 36.92$. Rounding to the nearest minute, we find that the correct time is 4:37.

The problem is open to a second interpretation, however. We could assume that the minute hand continues to move in the normal direction, while it is the hour hand that now moves counterclockwise. Interestingly, this has no effect on the final answer. In this case, when it is actually 4:00 the clock reads 8:00. After x minutes have elapsed, the minute hand will be pointing at minute mark x. The hour hand will be pointing at minute mark $40 - \frac{x}{12}$. This time the algebra reveals that

$$\begin{aligned} x &= 40 - \frac{x}{12} \\ \frac{13x}{12} &= 40, \end{aligned}$$

which is exactly the equation we found the first time.