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## Problem of the Week

### Solution Six

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My accurate, twelve-hour, circular clock has sixty, equally-spaced, minute marks, one at each minute. Identify all the times after 12:00 that my clock's continuously moving hour and minute hands point directly to consecutive minute marks.

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**SOLUTION:** The only such times are 1:12 and 9:48.

After 12:00, the hour hand points directly to a minute mark every twelve minutes. Thus, the only times we need to consider are those that are 12, 24, 36, 48, and 60 minutes past the hour.

Of course, 60 minutes past the hour can be dismissed out of hand. After sixty minutes we start a new hour, and at the start of any hour the hour and minute hands are certainly not pointing at consecutive minute marks.

Let us consider the 12. At twelve minutes past an hour, the hour hand will be pointing directly to the first minute mark past the hour. Thus, a time of 1:12 will satisfy the requirements of the problem, since the minute hand will then be pointing at the 12 minute mark and the hour hand will be pointing to the 11 minute mark. However, placing the minute hand on the 12 minute mark and the hour hand on the 13 minute mark does not correspond to a valid time.

We now continue systematically. At twenty-four minutes past the hour, the hour hand will be pointing to the second minute mark past the hour. This does not lead to any valid times that satisfy our requirements. At 4:24, this means that the hour and minute hands are pointing to minute marks that are two apart, but that is not close enough! So this case leads to no viable solutions.

At thirty-six minutes past the hour, the hour hand will be pointing to the third minute mark past the hour. This again leads to no viable solutions.

At forty-eight minutes past the hour, the hour hand will be pointing to the fourth minute mark past the hour. Thus, a time of 9:48 will work, since the minute hand is then pointing to the 48 minute mark, while the hour hand is pointing to the 49 minute mark.

Since this exhausts all the possibilities, we have found all of the valid answers.