
Problem of the Week

Solution Eight

At 4:00, the hands of a clock form a 120 degree angle. After how many minutes will the hands again form a 120 degree angle?

SOLUTION: The answer is 4:43 and $\frac{7}{11}$ minutes.

At 4:00, the hour hand makes an angle of 120 degrees with the line connecting the center of the clock to the 12. After x minutes have passed, the hour hand will have moved an additional $\frac{x}{2}$ degrees relative to this line segment. (This follows from a simple proportion, based on the fact that the hour hand plainly moves 30 degrees in 60 minutes.

For its part, the minute hand will have traversed an angle of $6x$ degrees in the same x minutes. Since the hour hand has a head start of 120 degrees, and since we want the minute hand to end another 120 degrees beyond the hour hand, we find that the conditions of the problem imply the equation

$$6x = 240 + \frac{x}{2}.$$

This is easily solved to find that

$$x = 43\frac{7}{11}$$

minutes, which leads to the answer given above.