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## Problem of the Week

### Solution Nine

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One fine day, sometime in the afternoon, the hour hand of Jane's clock started moving backward at the same speed at which it had previously been moving forward. The minute hand continued moving as it always had. Later that same day, Jane noticed that the hands of the clock displayed the time 6:35. This differed from the correct time by exactly five hours. At what time did the hour hand begin to move backward?

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**SOLUTION:** If we assume a twelve-hour clock, then there are two possible answers. The hour hand started moving backward either at 3:05 or 4:05 in the afternoon.

The key observation is that after the hour hand starts moving backward, the clock will only display a valid time on the half-hour. For example, suppose the correct time is 6:00. On a normal clock, at 6:30, the minute hand will point at the six and the hour hand will be halfway between the six and the seven. If the hour hand were moving backward instead, the hour hand would now be halfway between the five and the six, which would display the valid time of 5:30. But at, say, 6:15, the hour hand on the reverse clock will be closer to the six than to the five. Such a placement of the hands does not correspond to a valid time.

We conclude that at any valid time, the reverse clock can only appear to be a whole number of hours ahead or behind the correct time.

Suppose the reverse time is ahead of the real time. Then the time at which the reversal began must either have been  $6 - \frac{n}{2}$  hours ago, or  $12 - \frac{n}{2}$  hours ago. For example, suppose the reverse clock reads four hours ahead of the correct time. Then our formulas imply that the reversal began either two hours or ten hours earlier, both of which correspond to valid scenarios. If instead the reversed clock is behind the correct time, then the corresponding formulas are  $\frac{n}{2}$  or  $6 + \frac{n}{2}$ .

Currently, the clock reads 6:35, and this is either five hours ahead or five hours behind the correct time. If it is ahead, then our formulas imply the time of reversal was either  $\frac{7}{2}$  or  $\frac{19}{2}$  hours ago, giving initial times of 10:05 or 4:05. If it is behind, then the time of reversal was either  $\frac{5}{2}$  or  $\frac{17}{2}$  corresponding to initial times of 9:05 or 3:05. Since we were told the reversal started in the afternoon, we conclude that only 3:05 or 4:05 are valid solutions.