Problem of the Week
Number One
September 4, 2017

Biologists classify *Homo sapiens* among the tetrapods, a group of organisms that includes amphibians, reptiles, birds, and mammals. The term means "four-legged," or perhaps more generally "four-limbed." Those limbs might take very different shapes—wings and skinny legs on birds, for example, as contrasted with the four tree-trunks holding up an elephant—but invariably there are four nonetheless. This includes even the whales, whose tiny, vestigial femurs speak to an evolutionary history spent on land.

Each of these four limbs are almost invariably accompanied by five digits. We have "pentadactyl limbs," to use the biojargon, or "ten little piggies" in the terminology of an old children's song. Horses have one giant toe, but vestigial genes for making the other four are still there. Hummingbirds have only three, but these are just toes two, three, and four of an embryological process that includes the full complement of five.

Why five? Fossil tetrapods show specimens with everything from four to eight digits on their limbs. Perhaps five is simply the perfect number of digits. As our fishy ancestors took their first halting steps on land, maybe they found five digits, with two on each side providing support astride a central axis, was just the perfect arrangement. The inexorable process of natural selection would then assure that this arrangement persisted. Or maybe it is just an historical accident. Maybe sheer dumb luck explains the modern ubiquity of the pentadactyl limb.

Whatever the explanation, it seems plausible that our five-fingered hand, so perfectly suited to the manipulation of tools, made possible the evolution of our big brains. After all, of what use is a big brain if we lack the manual dexterity to do anything with it? Paleontologist Erik Jarvik writes, with reference to the ancient group of fishes from which we evolved, "We can say, with some justification, that it was when the basic pattern of our five-fingered hand for some unaccountable reason was laid down in the ancestors of the osteolepiforms that the prerequisite for the origin of man and the human culture arose."

You see the point? By considering both hands together we see that the number ten lies right at the heart of what it means to be human.

Folks, this is the tenth straight semester in which I will be bringing you these tasty mathematical morsels every week. For that reason, our theme this term is:

The Number Ten

Each week, I will bring you a problem that in some way involves the number ten. (Or possibly a power of ten. That's close enough.) Here is your first one. As always, we start with a fairly easy problem, but don't answer too quickly! It contains a trap for the unwary.

Let 10^a and 10^b represent the two, distinct, powers of ten nearest in value, but not equal to, the number 10^{2017}. What is $a + b$?

When you think you have the problem figured out, follow the instructions below.

*Submissions are due to Jason Rosenhouse by 5:00 on **Friday, September 8.** Solutions, complete with a brief explanation, should be written on the back of an official POTW handout. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the **upper right corner** of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Solutions will be posted at the POTW website:*

http://educ.jmu.edu/∼rosenhjd/POTW/Fall17/homepage.html