Problem of the Week
Number Two
September 11, 2017

We now continue with our ten-themed edition of POTW.

Mathematicians think it is fun to find numbers that can be arranged into intriguing geometrical shapes.

For example, collections of four, nine, or sixteen objects can be arranged into squares with respectively two, three, or four objects on a side. This leads to the concept of a square number, more commonly referred to as a perfect square. We shall consider 1 to be a perfect square, since one lonely object can be regarded as a square with one object on each side. The sequence of perfect squares begins with

$$1, 4, 9, 16, 25, 36, \ldots$$

Some numbers of objects can be arranged into triangles. For example, six objects can be formed into a triangle with three objects on the bottom row, two in the next row, and one on top. Picture six cheerleaders forming a human pyramid. Numbers that can be so arranged are said to be triangular numbers. The first few are

$$1, 3, 6, 10, 15, 21, \ldots$$

Triangular numbers are found by summing consecutive whole numbers, starting with 1.

What happens if we start summing triangular numbers? Do we thereby get a nice shape? Indeed we do! We get the so-called "tetrahedral numbers." (A tetrahedron is a pyramid resting on a triangular base.) Imagine that you are stacking oranges on a triangular base. Each level in the stack will be a triangular number. For example, we can imagine that six oranges, arranged in a triangle, are on the bottom. Then a triangle formed from three oranges would sit on top of them, with one more on top to complete the structure. The first few tetrahedral numbers are

$$1, 4, 10, 20, 35, 56, \ldots$$

Wait? Did you see what just happened? The number 10 appears on both lists! It is both triangular and tetrahedral. Aside from 1, which is really just an honorary figurate number, it is the smallest number to do so. (The only other numbers to appear on both lists are 120, 1540, and 7140).

A remarkable number, 10. Here is your second ten-themed problem for the term:

Let $x = 10^{10}$. For what value of y will 10^y equal the tenth root of 10^x?

Follow the instructions below.

Submissions are due to Jason Rosenhouse by 5:00 on ***Friday, September 15.*** *Solutions, complete with a brief explanation, should be written on the back of an official POTW handout. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the* ***upper right corner*** *of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Solutions will be posted at the POTW website:*

http://educ.jmu.edu/∼rosenhjd/POTW/Fall17/homepage.html