Problem of the Week
Number Five
October 2, 2017

We now continue our semester of ten-themed goodness.

While the number 10 is fascinating all by itself, we should not overlook its powers. Take 100, for example.

It is a perfect square, of course. But there are other sorts of figurate numbers as well. Back in POTW 2 we mentioned triangular numbers. For example, 6 is a triangular number, since we can place one dot on top, two beneath it, and then three on the bottom, much like a human pyramid. We could also mention pentagonal numbers, which—surprise!—are numbers of dots can be made into pentagons. (Google "pentagonal numbers" for the details of exactly what this means.)

Now, excluding 1 from our considerations, the smallest triangular number is 3, the smallest square number is 4, and the smallest pentagonal number is 5. And it just so happens that $(3, 4, 5)$ is a Pythagorean triple (meaning that $3^2 + 4^2 = 5^2$.)

Upon noticing this, how long do you think it took for mathematicians to wonder if there were other Pythagorean triples involving a triangle, a square, and a pentagon. Not long at all, that's how long! Currently the largest one known is $105, 100, 145$. Just so we're clear, 105 is a triangular number, 100 is a square, and 145 is pentagonal.

Boom! 100 in the strangest places.

Incidentally, 145 is a pretty fascinating number in its own right. Just in the time it took you to read that sentence, you probably noticed that

$$145 = 1! + 4! + 5! = 1 + 24 + 125,$$

where the exclamation point indicates a factorial (so that $4! = (4)(3)(2)(1)$, and so on.) The only other number greater than 2 with this property? Why, that would be 40585, *obviously*. (If you care to test that, keep in mind $0! = 1$ by definition.)

Here is this week's problem:

Twenty dancers line up, one behind the other. At every clang of the cymbals, the dancers in positions 10 and 20 dance forward into positions 1 and 2 respectively. The other 18 dancers keep their same positions relative to all of the others. What is the least number of times the cymbals must clang before the dancer originally in position 1 returns to that position?

When you think you have the problem figured out, follow the instructions below.

Submissions are due to Jason Rosenhouse by 5:00 on ***Friday, October 6.*** *Solutions, complete with a brief explanation, should be written on the back of an official POTW handout. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the* ***upper right corner*** *of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Solutions will be posted at the POTW website:*

http://educ.jmu.edu/∼rosenhjd/POTW/Fall17/homepage.html