
Problem of the Week

Number Six

October 16, 2017

More ten-themed goodness for you this week.

Many of the little numerical factoids I've been featuring in POTW this term have come from a wonderful book called *Those Fascinating Numbers* by Jean-Marie De Koninck, published by the AMS.

The book is simply a list of whole numbers, together with interesting facts about those numbers. For example, from this book you can learn that 26 is the smallest non-palindrome whose square is a palindrome: $26^2 = 676$. The next two numbers with this property are 264 and 307 as you can check for yourself.

Having already discussed 10 and 100, I went looking for the entry on 1000. To my surprise, there wasn't one. Apparently 1000 does not make De Koninck's standards for being fascinating. I *did* notice, however, that 1009 has a strange, ten-themed fact about it. It is the smallest prime number that can be written in any of the forms $x^2 + ky^2$, where k is any whole number between 1 and 10:

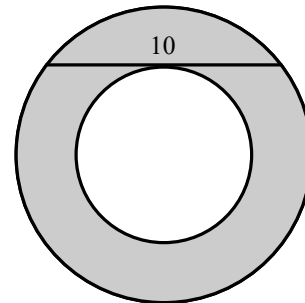
$$\begin{aligned}1009 &= 15^2 + 28^2 = 19^2 + 2(18)^2 \\ &= 31^2 + 3(4)^2 = 15^2 + 4(14)^2 \\ &= 17^2 + 5(12)^2 = 25^2 + 6(8)^2 \\ &= 1^2 + 7(12)^2 = 19^2 + 8(9)^2 \\ &= 28^2 + 9(5)^2 = 3^2 + 10(10)^2\end{aligned}$$

Also, 1009 (in addition to being the smallest four-digit prime number, of course), is the smallest number that can be written as the sum of three distinct positive cubes in two different ways:

$$1009 = 1^3 + 2^3 + 10^3 = 4^3 + 6^3 + 9^3$$

Now have a go at this week's problem:

In the pair of concentric circles shown below, a chord of the larger circle is drawn tangent to the smaller circle. This chord has length ten. Find the area of the annulus (shaded gray in the diagram).



When you think you have the problem figured out, follow the instructions below.

*Submissions are due to Jason Rosenhouse by 5:00 on **Friday, October 20**. Solutions, complete with a brief explanation, should be written on the back of an official POTW hand-out. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the **upper right corner** of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Solutions will be posted at the POTW website:*

[http://educ.jmu.edu/~rosenhjd/POTW/
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