Problem of the Week
Solution One

Let 10^a and 10^b represent the two, distinct, powers of ten nearest in value, but not equal to, the number 10^{2017}. What is $a + b$?

SOLUTION: We find that $a + b = 4031$.

The powers of ten closest in value to 10^{2017} are 10^{2016} and 10^{2015}. Thus $a = 2015$ and $b = 2016$, and their sum is 4031.

The trap here is in thinking that the nearest powers of ten would be 10^{2016} and 10^{2018}. But 10^{2018} is actually farther from 10^{2017} than is 10^{2015}. We compute:

$$10^{2018} - 10^{2017} = 10^{2017}(10 - 1) = 9 \times 10^{2017}$$
$$10^{2017} - 10^{2015} = 10^{2015}(100 - 1) = 99 \times 10^{2015}.$$

Since the first number has 2017 digits, while the second has 2016, we see that the first number is greater.

The principle is easier to see with smaller powers of ten. The two powers of ten closest to 1000 are 100 and 10, since each is closer to 1000 than is 10000.