Problem of the Week
Number Three
September 18, 2017

The square of the number $999{,}999{,}999$ is an 18-digit number. How many of these 18 digits are 9s?

SOLUTION: We find that 8 of the 18 digits are 9s.

The trick is to notice that

$$999{,}999{,}999 = 10^9 - 1.$$

It follows that

$$(999{,}999{,}999)^2 = (10^9 - 1)^2 = 10^{18} - 2(10)^9 + 1.$$

Factoring 10^9 from the first two terms leads to

$$10^9(10^9 - 2) + 1 = 10^9(999{,}999{,}998) + 1 = 999{,}999{,}998{,}000{,}000{,}001.$$

This number has eight 9s as claimed.