Problem of the Week

Solution Four

Find the smallest positive integer n with the property that 1500 divided by n leaves a remainder of 10.

SOLUTION: We find that $n = 149$.

Since n leaves a remainder of 10 when divided into 1500, there must be a whole number k with the property that

$$1500 = kn + 10.$$

In other words, 1500 is ten more than some multiple of n.

It follows that $kn = 1490$, which in turn implies that n must be a factor of 1490. We now observe that

$$1490 = 2 \times 5 \times 149.$$

The numbers 2 and 5 are factors of 1500, so they do not leave a remainder of 10 when divided into 1500. The next smallest factor of 1500 is 149, and we see that this works.