Problem of the Week
Solution Five

Twenty dancers line up, one behind the other. At every clang of the cymbals, the dancers in positions 10 and 20, dance forward into positions 1 and 2 respectively. The other 18 dancers keep their same positions relative to all of the others. What is the least number of times the cymbals must clang before the dancer originally in position 1 returns to that position?

SOLUTION: The cymbals must clang twenty times.

Each of the first five clangs of the cymbal cause the first dancer to advance two places. That is, the original first dancer's position after each of the first five clangs is given by:

$$3, 5, 7, 9, 11.$$

Since this dancer is now ahead of dancer ten, the next nine clangs will only advance him one place. His position after each of the next nine clangs is:

$$12, 13, 14, 15, 16, 17, 18, 19, 20.$$

So, after fourteen clangs he is in position 20. The next clang will advance him to position 2. Four more clangs lead to

$$4, 6, 8, 10.$$

The next clang is number twenty, and it will finally push him back to the first position.