
Problem of the Week

Solution Nine

When Professor Smith's will was read upon his death, it was found that he bequeathed 10 of his books to his students Anne, Bill, and Carl. The three students were to choose from among his books in a particular order. Smith's wishes with regard to the order were captured in the following three statements:

1. No person who has seen me in a green tie is to choose before Anne.
2. If Bill was not in Oxford in 1920, then the first chooser never lent me an umbrella.
3. If Bill or Carl has second choice, then Carl comes before the one who first fell in love.

Unfortunately, none of the students could remember any of the relevant facts. However, knowing Professor Smith's fondness for economy of information, they reasoned he would not include superfluous statements. They then realized that, assuming that every statement provided essential information toward the solution, there is only one possible order. Can you work out that order?

SOLUTION: The only viable ordering is Anne, Carl, Bill.

The key observation is that every statement provides essential information towards the solution. Any proposed ordering that entails that

one of the statements provides nothing useful can be discarded.

In order for (1) to provide us with information, it must be that at least one of Bill and Carl has seen Smith in a green tie. If neither has seen him in a green tie, then the statement is redundant. We deduce that Anne cannot be the third chooser, since she must be followed by those who saw Smith in a green tie.

Move on to statement (2). If Bill was in Oxford in 1920 then (2) provides no new information about the order, so we can say that Bill was not in Oxford in 1920. Moreover, if no one lent Smith an umbrella, the statement is superfluous. So, someone lent Smith an umbrella.

Who lent Smith an umbrella? If Anne lent the umbrella, then from (2) we conclude that Anne is not first. We know Anne is not last from (1), so this would put Anne second. But if Anne is second, then (3) contributes nothing to the solution. It follows that Anne did not lend the umbrella.

If both Bill and Carl lent Smith an umbrella, then from (2) we have that Anne is first, and from (3) Carl is second and Bill is third. But this solution makes (1) superfluous. We conclude that either Bill or Carl lent Smith an umbrella, but not both.

Likewise, if both Bill and Carl saw Smith in a green tie, then from (1) Anne is first and (2) is now superfluous. So either Bill or Carl saw Smith in a green tie but not both.

Let us continue: Suppose that Bill saw Smith in a green tie *and* lent him an umbrella. Then from (1) we know that Bill cannot be first, which if true would make (2) superfluous. So, if Bill saw Smith in a green tie, then he cannot have lent him an umbrella. This implies that

Carl lent the umbrella. Likewise, if Carl saw Smith in a green tie, the same logic applies and Bill must have lent Smith an umbrella.

In both of these cases, Anne must be first. And, if this is the case, from (3) we see that Bill must be the one who first fell in love. This leads to the final ordering that Anne chose first, then Carl, then Bill, as claimed.